FISK

A Course in Alternating Currents

for Senior Students

Electrical Engineering

E. E.

1914

A COURSE IN ALTERNATING CURRENTS FOR SENIOR STUDENTS

BY

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B. S. University of Illinois, 1909

M. S. University of Illinois, 1913

THESIS

Submitted in Partial Fulfillment of the Requirements for the

Degree of

ELECTRICAL ENGINEER

IN

THE GRADUATE SCHOOL

OF THE

UNIVERSITY OF ILLINOIS

1914

UNIVERSITY OF ILLINOIS

THE GRADUATE SCHOOL

May 1, 1914

I HEREBY RECOMMEND THAT THE THESIS PREPARED BY

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A COURSE IN ALTERNATING CURRENTS

FOR SENIOR STUDENTS

I

INTRODUCTION

In the teaching of Alternating Current Theory to senior students it is necessary to apply the fundamental principles, which the student has been taught, to calculations of electrical machines.

In the majority of text books, which are at all adapted to senior students, the tendency is to cover a large number of machines and to give only a small amount of theory pertaining to each machine.

It is the purpose of this thesis to present a course suitable for senior Electrical Engineering students, which will treat fully the theory of a few pieces of electrical apparatus.

In this course the theory is gradually given to the student by a series of problems and lectures and may be divided into the following general classifications.

II
CIRCUITS

Practically all electrical machines may be studied and understood if we can obtain their electrical constants and apply them to an equivalent electrical or magnetic circuit.

It therefore follows from the above statement that the electrical engineer should become familiar with the solution of complex circuits.

The first few problems are given in order to review the work which the student has had during the previous year and to prepare him for the more difficult circuits which are to follow.

Given $E_0 = 100$, $r_1 = 3$, $x_1 = 4$, $r_2 = 6$, $x_2 = 8$.

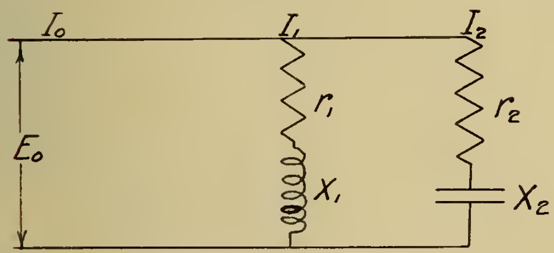


Fig. 1.

Find I_1, I_2 and I_0.

A current vector diagram is shown in Fig. 2.

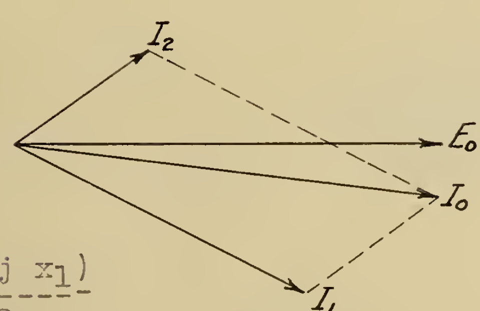


Fig. 2.

General solution.

$$\dot{I}_0 = \dot{I}_1 + \dot{I}_2$$

$$\dot{I}_1 = \frac{E_0}{r_1 + Jx_1} = \frac{E_0}{\dot{Z}_1} = \frac{E_0(r_1 - j\,x_1)}{r_1^2 + x_1^2}$$

$$= \frac{E_0 \, r_1}{r_1^2 + x_1^2} - \frac{J \, E_0 \, x_1}{r_1^2 + x_1^2} = E_0 (g_1 - J \, b_1) = E_0 \, Y_1 \, .$$

Where $g_1 - J \, b_1$ is expressed as follows,

$$g_1 = \frac{r_1}{r_1^2 + x_1^2} \quad \text{and} \quad b_1 = \frac{x_1}{r_1^2 + x_1^2}$$

Hence $I_1 = \frac{E_0}{Z_1} = \frac{E_0}{r_1 + J \, x_1} = E_0 \, (g_1 - J \, b_1) = E_0 \, Y_1$.

For inductive reactance and resistance in series

$$Z = r + J \, x,$$

and for a circuit of capacity reactance and resistance in series,

$$Z = r - J \, x,$$

or for the general expression we may use only the form,

$$Z = r + J \, x,$$

and for inductive reactance this is correct but for capacity reactance we will have to write,

$$Z = r - J \, (-x).$$

Hence we may always write

$$Z = r \pm J \, x,$$

using the + sign with inductive reactance and the - sign with capacity reactance.

Now let us investigate Y.

$$Y = \frac{1}{Z} = g - J \, b,$$

and for inductive reactance we have

$$Y = \frac{1}{Z} = \frac{1}{r + Jx} = \frac{r - J \, x}{r^2 + x^2} = \frac{r}{r^2 + x^2} - \frac{J \, x}{r^2 + x^2},$$

and if

$$g = \frac{r}{r^2 + x^2} \quad \text{and } b = \frac{x}{r^2 + x^2}$$

we have $\underset{\cdot}{Y} = g - J\,b$ for inductive reactance.

Again,

$$\underset{\cdot}{Y} = \frac{1}{\underset{\cdot}{Z}} = \frac{1}{r - Jx} \text{ for capacity circuits}$$

$$\frac{1}{r - Jx} = \frac{r + Jx}{r^2 + x^2} = \frac{r}{r^2 + x^2} + \frac{Jx}{r^2 + x^2}$$

again letting

$$\frac{r}{r^2 + x^2} = g \quad \text{and } \frac{x}{r^2 + x^2} = b$$

we have $\underset{\cdot}{Y} = g + Jb$ for capacity.

Now for the general expression always write,

$$\underset{\cdot}{I} = \frac{E}{\underset{\cdot}{Z}} = \frac{E}{r + Jx}$$

If x is inductively reactive the expression is correct as above; if capacity, x must be preceeded by the - sign.

Again

$$\underset{\cdot}{Y} = g + Jb.$$

If the circuit contains inductance then b must be preceeded by the - sign and if the circuit contains capacity the expression is correct as indicated above.

Problem solution.

$$\underset{\cdot}{I}_1 = \frac{E_o}{r_1 + Jx_1} = \frac{E_o(r_1 - Jx_1)}{r_1{}^2 + x_1{}^2} = E_o\, \underset{\cdot}{Y}_1 = E_o\,(g_1 - Jb_1)$$

$$\underset{\cdot}{I}_2 = \frac{E_o}{r_2 + J(-x_2)} = \frac{E_o}{r_2 - Jx_2} = \frac{E_o\ r_2 + Jx_2}{r_2^2 + x_2^2} = E_o\ (g_2 + Jb_2)$$

$$\underset{\cdot}{I}_o = E_o\ (\underset{\cdot}{Y}_1 + \underset{\cdot}{Y}_2) = E_o\left[(g_1 + g_2) + (Jb_2 - Jb_1)\right]$$

$$= E_o\left[(g_1 + g_2) + J\ (b_2 - b_1)\right]$$

$$g_1 = \frac{r_1}{r_1^2 + x_1^2} = \frac{3}{9 + 16} = \frac{3}{25} \qquad g_2 = \frac{r_2}{r_2^2 + x_2^2} = \frac{6}{36 + 64} = \frac{6}{100}$$

$$b_1 = \frac{x_1}{r_1^2 + x_1^2} = \frac{4}{9 + 16} = \frac{4}{25} \qquad b_2 = \frac{x_2}{r_2^2 + x_2^2} = \frac{8}{36 + 64} = \frac{8}{100}$$

$\underset{\cdot}{I}_1 = E_o\ (g_1 - Jb_1)$ - sign indicates lagging current.

$\underset{\cdot}{I}_1 = 100\ (\frac{3}{25} - J\ \frac{4}{25})$ and $I_1 = 100\sqrt{(\frac{3}{25})^2 + (\frac{4}{25})^2} = 20$ amp.

$\underset{\cdot}{I}_2 = E_o\ (g_2 + Jb_2)$ + sign indicates leading current.

$\underset{\cdot}{I}_2 = 100\ (\frac{6}{100} + J\ \frac{8}{100})$ and $I_2 = 100\sqrt{(\frac{6}{100})^2 + (\frac{8}{100})^2} = 10$ amp.

$$\underset{\cdot}{I}_o = \underset{\cdot}{I}_1 + \underset{\cdot}{I}_2$$

$$= E_o\left[(g_1 + g_2) + J\ (b_2 - b_1)\right]$$

$$= E_o\sqrt{(g_1 + g_2)^2 + (b_2 - b_1)^2}$$

$$g_1 + g_2 = \frac{18}{100}$$

$$(b_2 - b_1) = -\left(\frac{8}{100}\right)$$

and indicates that the total current is lagging the e.m.f. E_0.

$$I_0 = 100\sqrt{(\frac{18}{100})^2 - (-\frac{8}{100})^2} \quad \text{- sign here}$$

only indicates the direction the real value of I_0 .

$$I_0 = 100\sqrt{(\frac{18}{100})^2 + (\frac{8}{100})^2} = 19.5 \text{ amperes.}$$

The power factor etc. will be solved for later.

Problem 1a.

Given the circuit as indicated in Fig. 3, draw the vector diagram for same, using e as a reference vector and assume an inductive load.

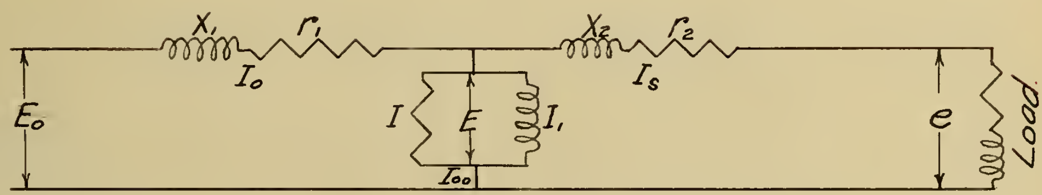


Fig. 3.

Solution of problem 1a.

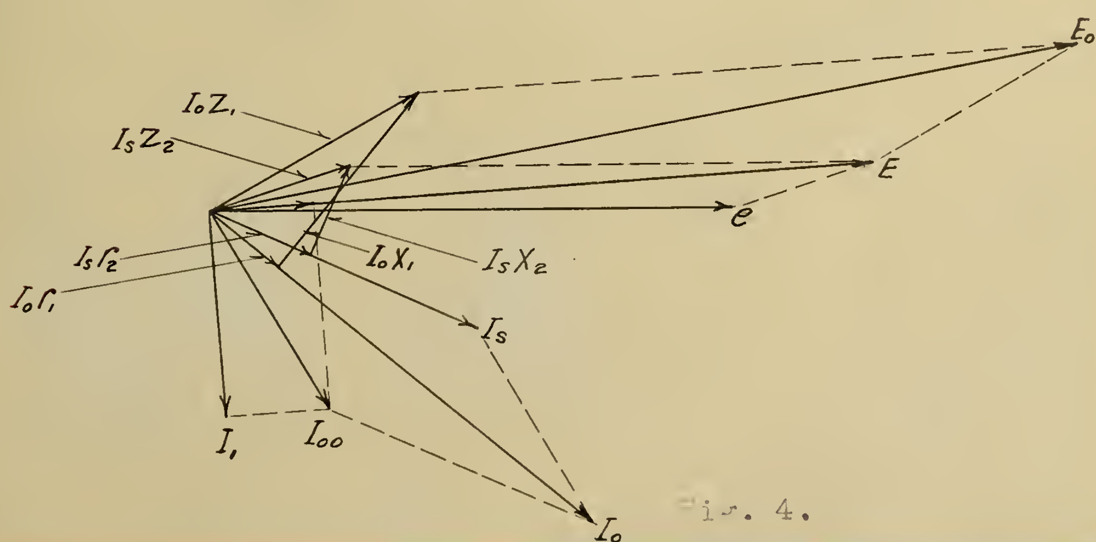


Fig. 4.

Problem #2.

Given a circuit as indicated in Fig. 5.

(a) Find the current flowing in the circuit, the power consumed, and power factor.

(b) Calculate at what frequency the maximum current will flow.

(c) When maximum current is flowing calculate the voltage across each part of the circuit.

(d) Calculate the maximum energy stored in the condenser at resonance.

(e) Calculate the maximum energy stored in the inductance at resonance.

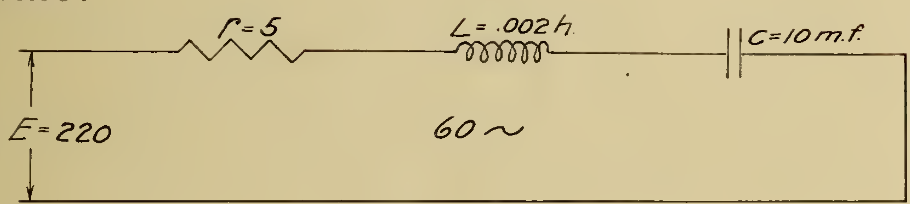


Fig. 5.

Solution of Problem #2.

$$\dot{I} = \frac{E}{\dot{Z}} = \frac{E}{r + j\,(x_L - x_c)}$$

$$I = \frac{E}{\sqrt{r^2 + \left(2\pi fL - \frac{1}{2\pi fc}\right)^2}}$$

$$x_L = 2\pi fL = 2 \times 3.14 \times 60 \times .002 = .754$$

$$x_c = \frac{1}{2\pi fc} = \frac{1}{2 \times 3.14 \times 60 \times .00001} = 265.5$$

$$E = 220 \qquad r^2 = 25$$

$$\left(2\pi fL - \frac{1}{2\pi fc}\right)^2 = (-\ 264.746\ )^2 = 70,000$$

$$Z = \sqrt{25 + 70,000} = 265.$$

From the above it is evident that both r and $2\pi fL$ or x_L might have been neglected without introducing any apparent error.

$$I = \frac{E}{Z} = \frac{220}{265} = .831 \text{ amperes flowing in the circuit.}$$

Power consumed $= I^2R = \overline{.831}^2 \times 5 = 3.45$ watts.

Power factor $= \cos\theta = \dfrac{W}{E\ I} = \dfrac{3.45}{220 \times .831} = .01876$

The maximum current will flow when the frequency is such that

$$2\pi fL = \frac{1}{2\pi fc} \qquad \text{or } f = \frac{1}{2\pi}\sqrt{\frac{1}{Lc}}$$

$$f^2 = \frac{1}{4\pi^2 Lc} = \frac{1}{4 \times 3.14 \times 3.14 \times .002 \times .00001}$$

$$f^2 = \frac{1}{.0000007888}$$

$$f^2 = 1,268,000$$

$$f = 1126 \text{ cycles per second.}$$

At this frequency $x_L = 2\pi\ 1126 \times .002 = 14.14$ ohms and

$$-x_c = \frac{1}{2\pi fc} = \frac{1}{2\pi \times 1126 \times .00001} = 14.14 \text{ ohms.}$$

If $\dfrac{1}{2\pi fc} = 2\pi fL$ then $I = \dfrac{E}{r}$

or current $= \dfrac{220}{5} = 44$ amperes.

Voltage across the resistance $= Ir = 44 \times 5 = 220.$

Voltage across the reactance = IX_L = 44 x 14.14 = 622

Voltage across the capacity = IX_c = 44 x 14.14 = -622

Maximum energy stored in the condenser at the above frequency is

$$E_n = \sqrt{\frac{2}{2}}\ C\ E^2 = \sqrt{\frac{2}{2}} \times .00001 \times \overline{622}^2 = 2.73 \text{ watt seconds.}$$

and the maximum energy stored in the inductance is

$$E_n = \sqrt{\frac{2}{-2}}\ L\ I^2 = \sqrt{\frac{2}{-2}} \times .002 \times \overline{44}^2 = 1.37 \text{ watt seconds.}$$

Problem #3.

Prove that $x = 2\pi fL$

Where x = reactance in ohms

f = frequency in cycles per second

L = the inductance in henrys

Assume that the current follows a sine wave.

Proof:

$$i = I \sin \omega t$$

$$e = ir + L \frac{di}{dt},$$

if there is no resistance in the circuit e impressed = e induced and,

$$e = L \frac{di}{dt} \qquad \frac{di}{dt} = \omega I \cos \omega t$$

$e = L\ \omega\ I \cos \omega t$ but $e = ix$ and $E(max) = I\ X$

$e = E\ (max)$ when $\cos \omega t = 1$ or $E = L\ \omega\ I$

$$\frac{E}{I} = x = L\ \omega = 2\pi fL$$

Problem #4.

Prove that the current in an inductive circuit lags behind the impressed e. m. f.

Proof:

$$e = ir + L\frac{di}{dt}$$

Again,

$$i = I \sin \omega t$$

$$\frac{di}{dt} = \omega I \cos \omega t = \omega I \sin (\omega t + 90)$$

$$e = L\frac{di}{dt} = L \omega I \cos \omega t \quad \text{and for constant } f, L, \text{ and } I$$

$$i = K \sin \omega t \quad \text{and}$$

$$e = K_1 \cos \omega t$$

$$e = K_1 \sin (\omega t + 90)$$ and hence it is evident from the

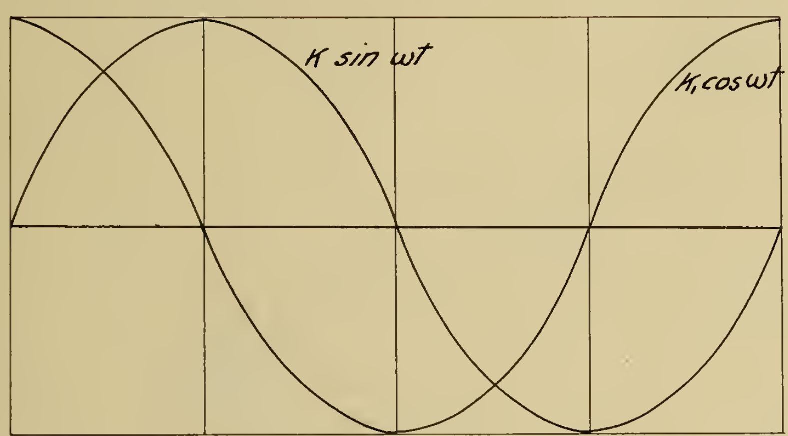


Fig. 6.

equation and also from Fig. 6, that values of i are 90° behind the corresponding values of e or as is usually stated the current lags the e. m. f. by 90 time degrees.

Problem #5.

Prove that with capacity in the circuit the current is leading by 90 time degrees.

Let us assume that the charge on the condenser will vary harmonically then,

$q = Q \sin \omega t.$

Now $q = c\ e$ or $e = \frac{q}{c}$

But $i = \frac{dq}{dt} = \omega Q \cos \omega t = \omega Q \sin (\omega t + 90)$

Therefore it is again seen that e and i are 90° out of phase and that i is leading, this is true since e must be the voltage across the terminals of the condenser at all times to hold the charge q.

Problem #6.

Prove that $E_n = 1/2\ L\ i^2$ where E_n = the energy stored in the reactance

$$e = ir + L \frac{di}{dt} = \text{voltage at any instant.}$$

Let i = the current at any instant.

$$ei = i^2 r - L\ i \frac{di}{dt} = \text{input to circuit at any instant.}$$

Multiplying by dt,

$ei\ dt = i^2 r\ dt + L\ i\ di$ = input to the circuit at any instant.

$$\int_{t_o}^{t_1} ei\ dt = \text{total input to the circuit}$$

$$= \int_{t_o}^{t_1} i^2 r\ dt + \int_{t_o}^{t_1} L\ i\ di$$

$$\int_{t_o}^{t_1} ei\ dt = \text{watt seconds total input}$$

$$\int_{t_o}^{t_1} i^2 r\ dt = \text{watt seconds} = \text{energy dissipated}$$

in the resistance.

$$\int_{t_0}^{t_1} Li\, di = \left[\frac{1}{2} L i^2\right]_{t_0}^{t_1} = \text{energy stored in the inductance in watt seconds or joules.}$$

Or the energy stored when $i = i_o$ will be

$$\frac{1}{2} Li_o^2 \text{ watt seconds.}$$

By the same method the energy stored in a condenser may be found to be

$$E_n = 1/2 \quad C E^2$$

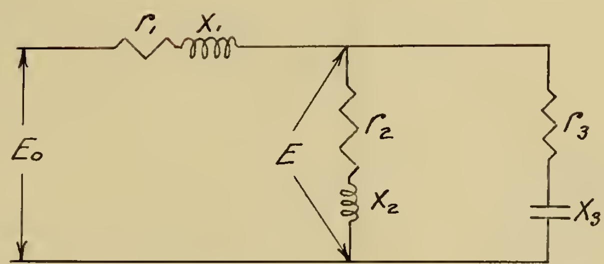


Fig. 7.

Problem #7.

Given $E = 100$

$$Z_{.1} = 2 + j4$$

$$Z_{.2} = 3 - 14$$

$$Z_{.3} = 5 + j(-5)$$

Find E_o and I. Draw complete vector diagram to approximate scale.

Problem #8.

Prove that $X_c = \frac{1}{2\pi fc}$. Specify the conditions, if any, that are necessary for this relation to be true.

Problem #9.

A sine wave of E. M. F. is impressed upon a resistance, a simple inductance and a simple capacity in parallel. By means of waves and a table show how the several currents combine to make the total current. Also draw the vector diagram. Assume,

$$i_R = i_L = 2\ i_c$$

Solution of Problem #7.

$$Y_2 = \frac{1}{Z_2} = \frac{1}{3 + j4} = \frac{3 - j4}{25} = .12 - j\ .16$$

$$Y_3 = \frac{1}{Z_3} = \frac{1}{5 - j5} = \frac{5 + j5}{50} = .10 + j\ .10$$

$$Y_4 = Y_2 + Y_3 = .22 - j\ .06$$

$$I = E\ Y_4 = 100\ (.22 - j\ .06) = 22 - j6$$

$$I = \sqrt{\overline{22}^2 + \overline{6}^2} = 22.8$$

$$E_1 = I\ Z_1 = (22 - j6)\ (2 + j4) = 44 - j12 + j88 + 24$$

$$E_1 = 68 + j76$$

$$E_o = E + E_1 = 100 + 68 + j76$$

$$E_o = 168 + j76$$

$$E_o = \sqrt{\overline{168}^2 + \overline{76}^2} = 184 \text{ volts.}$$

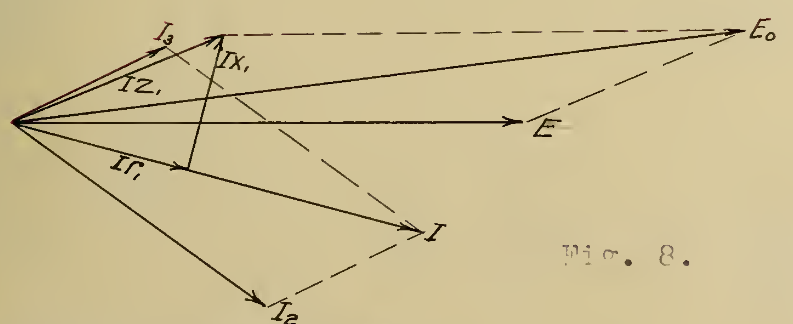


Fig. 8.

Solution of Problem #8.

Assume that a harmonic E. M. F. is applied to the terminals of a condenser.

That is $e = E \sin \omega t$

$$q = c\,e. \quad \text{But } i = \frac{dq}{dt}.$$

Hence $i = \dfrac{dq}{dt} = C\,\dfrac{de}{dt} = c\,\omega\,E \cos \omega t$

$$I_{max} = c\,\omega\,E \quad \text{or } E_{max} = \frac{I_{max}}{\omega c}$$

But

$$E_{max} = I_{max}\,X_c = \frac{I_{max}}{\omega c}$$

Hence

$$X_c = \frac{1}{\omega c} = \frac{1}{2\pi f c}$$ Based on a sine wave of E. M. F. and current.

For solution of Problem #9, see Table I and Curve Sheet #1, page 20. In this Table,

$e = E \sin \theta$ = the impressed voltage.

$i_R = I_r \sin \theta$ = the current in the resistance circuit.

$i_L = I_L \sin (\theta - 90^\circ)$ = the current in the inductive circuit.

$i_c = I_c \sin (\theta + 90^\circ)$ = the current in the capacity circuit.

$i = i_R + i_L + i_c$ = the total current in the line.

TABLE I

θ	e	i_R	i_L	i_c	i
0	0	0	-1.00	.50	-.50
10	.2075	.173	-.984	.492	-.319
20	.411	.342	-.939	.469	-.128
30	.600	.50	-.866	.433	.067
40	.770	.642	-.766	.383	.259
50	.920	.766	-.642	.321	.445
60	1.040	.866	-.50	.25	.616
70	1.126	.939	-.342	.171	.768
80	1.180	.984	-.173	.086	.898
90	1.200	1.00	0	0	1.00
100	1.180	.984	.173	-.086	1.061
110	1.126	.939	.342	-.171	1.11
120	1.04	.866	.500	-.25	1.116
130	.920	.766	.642	-.321	1.097
140	.770	.642	.766	-.383	1.025
150	.600	.50	.866	-.433	.933
160	.410	.342	.939	-.469	.812
170	.2075	.173	.984	-.492	.665
180	0	0	1.00	-.50	.50
190	-.2075	-.173	.984	-.492	.319
200	-.410	-.342	.939	-.469	.128
210	-.600	-.500	.866	-.433	-.067
220	-.770	-.642	.766	-.383	-.259
230	-.920	-.766	.642	-.321	-.445
240	-1.040	-.866	.50	-.25	-.616
250	-1.126	-.939	.342	-.171	-.768
260	-1.180	-.984	.173	-.086	-.898
270	-1.2	-1.00	0	0	-1.00

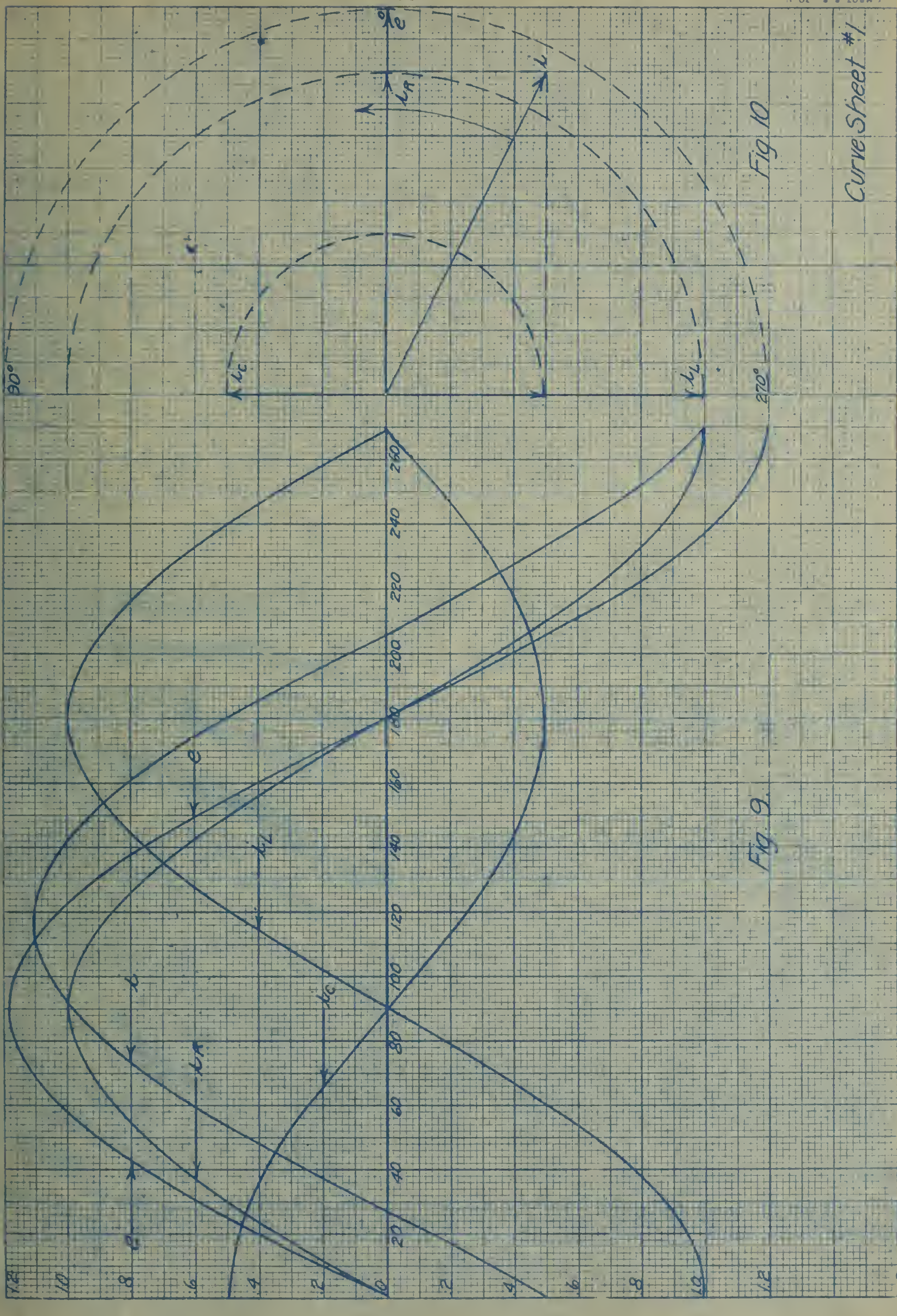


Fig. 9.

Fig 10

Curve Sheet #1

Problem #10.

Prove $E = 4.44 \times f \times \phi \times N \times 10^{-8}$.

Solution of Problem #10.

When a coil revolves in a magnetic field such as is produced by an electric generator the flux inclosed by the coil will vary from instant to instant very nearly as a sine wave, or expressed mathematically, $\varphi = \underline{\varphi} \sin \omega t$, where φ = the flux inclosed at any instant, and $\underline{\varphi}$ = maximum flux inclosed by the coil, this flux is the total flux per pole.

Since $e = N \frac{d\varphi}{dt}$ and $\varphi = \underline{\varphi} \sin \omega t$, we have

$$\frac{d\varphi}{dt} = \omega \underline{\varphi} \cos \omega t$$

and

$e = N \omega \underline{\varphi} \cos \omega t$ when $\cos \omega t = 1$ $e = E_{max}$

and

$E_{max} = N \omega \underline{\varphi} = 2\pi f N \underline{\varphi}$ (abvolts)

But

$$\frac{E_{max.}}{\sqrt{2}} = E_{eff} = \frac{2\pi}{\sqrt{2}} f N \underline{\varphi} \text{ (abvolts)}$$

$$E_{eff} = 4.44 f N \underline{\varphi} 10^{-8}.$$

This formula is equally true for the transformer, where of course $\underline{\varphi}$ is the total maximum flux in the core.

Problem #11.

An A. C. Generator has 50 turns on the armature; the machine has 4 poles and runs at 900 R. P.M. If the terminal voltage is 50; what must be the flux per pole?

Solution of Problem #11.

$$E_{eff} = 4.44 \times f \times \Phi \times N \times 10^{-8}$$

$$E = 50 \quad f = 30 \quad N = 50$$

$$\Phi = \frac{50 \times 10^8}{4.44 \times 30 \times 50} = 750{,}000 = \text{flux per pole.}$$

Problem #12.

A coil 50 cms. long and 25 cms. in diameter has 100 turns and is connected to a 100 volt circuit at 60 cycles.

Neglecting the resistance of the coil and the reluctance of the magnetic path outside the coil; what will be the current when the coil is connected to the line?

Solution of Problem #12.

$$E = 4.44 \times f \times \Phi \times N \times 10^{-8}$$

$$100 = 4.44 \times 60 \times \Phi \times 100 \times 10^{-8}$$

$$\Phi = 375.000 \text{ lines.}$$

$$\text{Area of coil} = \frac{\pi D^2}{4} = \frac{\pi \times 25 \times 25}{4} = 491 \text{ sq. cm.}$$

The density inside the coil will be

$$\frac{375.000}{491} = 764 \text{ lines per sq. cm.}$$

Since for an air cored coil $\mu = 1$ we have

$$\beta = \mu H \quad \text{or } 764 = \frac{.4\pi N i}{l} \quad \text{and since } N = 100$$

$$764 = \frac{.4\pi \times 100 \times i}{50}$$

$$\frac{764 \times 50}{.4\pi \times 100} = i = 304.4 \text{ amperes.}$$

But since Φ is the maximum flux β must be the maximum flux density and therefore i must be the maximum current.

$\frac{i}{\sqrt{2}}$ = effective value of current that will flow through the coil or

$$I_{eff} = \frac{i}{\sqrt{2}} = \frac{304.4}{\sqrt{2}} = 215.8$$

III

GENERAL QUESTIONS ON THE CONSTANT POTENTIAL TRANSFORMER

(a) Why can we use a single circuit to represent the transformer?

(b) What values must be used for primary and secondary resistance and reactance?

(c) How do these quantities vary with the load?

(d) What is meant by leakage flux, useful flux by reactance?

(e) What is meant by equivalent resistance and reactance?

(f) How are the resistance and reactance found for a given transformer.

(g) Draw the equivalent circuit for a transformer.

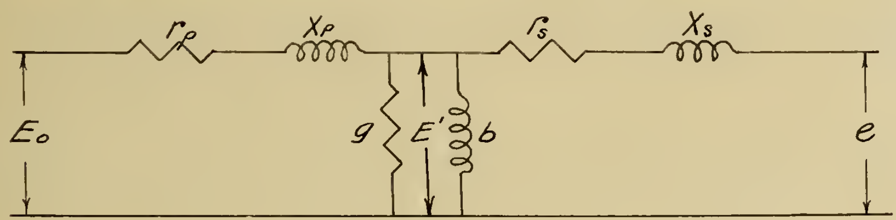


Fig. 11.

(h) What current is assumed to flow through g, and what current through b?

(i) How may g and b be measured for an actual transformer?

(j) How does the value of x_1 and x_2 compare with the % IX of secondary?

(k) How does the % IX primary compare with the % IX of secondary?

(l) Why not connect g and b as shown below?

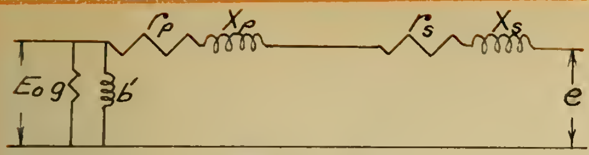


Fig. 12.

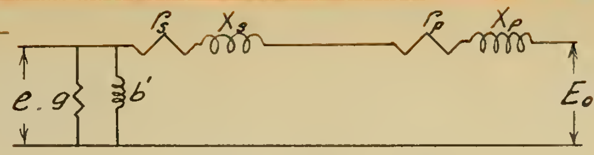


Fig. 13.

(m) What are equivalent resistance and reactance? Is it strictly correct to use the value of equivalent resistance and reactance when calculating the preformance of a transformer? Discuss the effect of resistance and reactance on the operation of a transformer.

(n) What is the effect of leakage flux? What is meant by self induction and mutual induction?

(o) Could a transformer be built with zero leakage flux?

(p) Draw sketch to illustrate the placing of coils to produce a large leakage flux, also to produce as little leakage flux as possible.

(q) Sketch the path of the leakage flux.

(r) Why does the current in the primary increase when load is added to the secondary?

(s) Draw the vector diagram of the transformer, starting with a ratio of 2:1, assume E_s, and that the secondary current is in phase with the secondary terminal voltage.

(t) Place on this diagram the various magneto motive forces.

(u) Why use leakage flux to calculate leakage reactance?

(v) Does leakage flux vary with the current?

(w) Does IX vary with current?

Note: Since $L = \frac{n\varphi}{I}$ it follows that $\frac{\varphi}{I}$ is a constant and hence $x = 2\pi fL$ = a constant.

(x) What flux can vary directly with current?

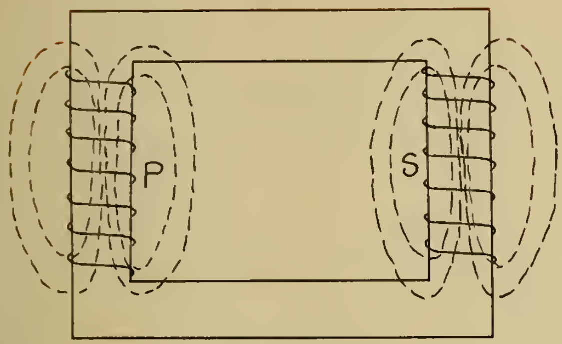


Figure 14 represents the path of the leakage flux.

Fig. 14.

Transformer calculations.

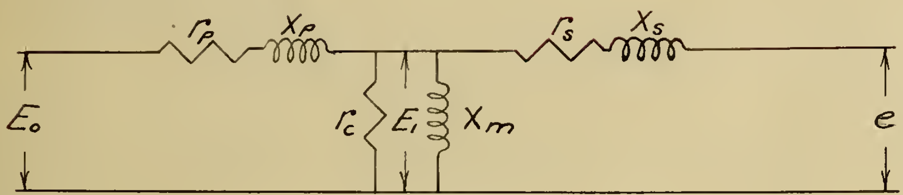


Fig. 15.

GENERAL SOLUTION OF TRANSFORMER CIRCUITS

The general solution is usually made assuming the secondary current $I_s = i + j\,i_1$ without regard to whether $I_s = i + j\,i_1$, represents leading or lagging current. From the above diagram it is evident that $E_1 = e + I_s Z_s$. Now let us assume that $Z_s = r_s + j\,x_s$ then

$$
\begin{aligned}
E_1 &= e + (i - j\,i_1)(r_s + j\,x_s) \\
&= e + ir - i_1 x_s - j(i_1 r_s + i x_s) \\
&= a + j\,b.
\end{aligned}
$$

The following diagram represents these terms assuming a lagging current.

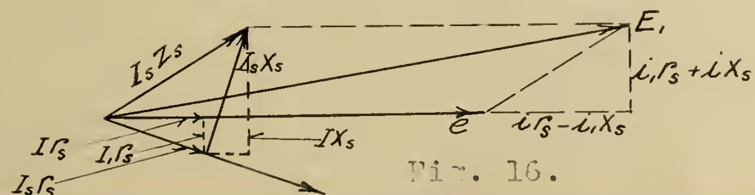


Fig. 16.

Now from Fig. 16 it is evident that the line representing $i\ r_s \mp i_1\ x_s$ is the sum of these two quantities and not their differences, hence it is evident that for a lagging current i_1 must be represented as being negative and by substitution we will have $i\ r_s + i_1\ x_s$ which is correct. Again from the diagram it is seen that the vector represented as $i_1\ r_s + i\ x_s$ is not the sum of these two quantities but their difference and hence again i_1 must be represented as negative when a lagging current is used and as + when a leading current is used. This is made evident and easy to remember by a study of the following diagram for lagging current.

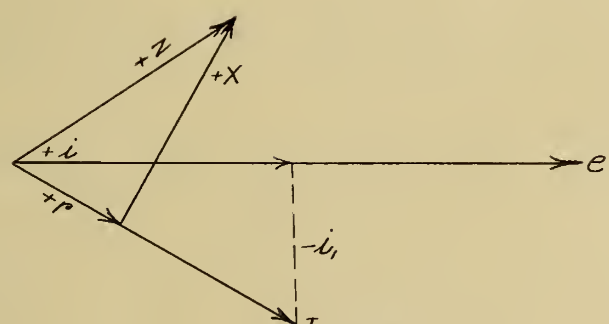


Fig. 17.

As shown above,

$$E_1 = a + j\ b \quad \text{where } a = e + ir_s - i_1x_s \text{ and}$$
$$b = i_1r_s - i\ x_s$$

The branch circuits represent the core loss current and the magnetizing current, their vector sum being the exciting current of the transformer. These currents are, of course, produced by the voltage E_1.

Representing this resistance as r_c and the reactance X_m it is evident that the current through r_c will be

$$I_c = \frac{E_1}{Z_c} = \frac{E_1}{r_c + j\,x_c} = \frac{E_1\,(r_c - jx_c)}{r_c^2 + x_c^2}$$

$$= E_1 \quad \frac{r_c}{r_c^2 + x_c^2} - j\,\frac{x_c}{r_c^2 + x_c^2}$$

$$= E_1\;(g_c - J\,b_c^1) \quad \text{but since}$$

$x_c = 0$

$$I_c = E_1\,g_c \quad \text{where } g = \frac{1}{r_c}$$

$$I_m = \frac{E_1}{r_m - j\,x_m} = \frac{E_1\,(r_m - jx_m)}{r_m^2 + x_m^2}$$

$$= \frac{E_1\,r_m}{r_m^2 + x_m^2} - J\,\frac{E_1\,x_m}{r_m^2 + x_m^2}$$

$I_m = E_1(g_m - j\,B_m 1)$. Now if we hold to the general rule, keeping the + sign before all terms preceded by j, it is evident we must write for I_m.

$I_m = E_1\left[g_m + j\,(-b_m 1)\right]$ that is for lagging current the term represented by b^1 must be negative and for leading current it must be positive, therefore since b^1 represents the susceptance of the transformer it will be negative for all loads.

Then the total exciting current I_{oo} will be, since g_m is zero,

$$I_{oo} = E_1\left[(g_c - j\,(-b_m 1)\right]$$

or for any current

$$I_{oo} = E_1\;(g + j\,b^1)$$

g being positive always and b^1 negative for lagging current and positive for leading current.

$$\underset{\cdot}{I}_{oo} = (a + jb)\ (g + j\ b^1)$$

$$= ag - b\ b^1 + j\ (bg + a\ b^1)$$

$$= f + j\ h \qquad \text{where}$$

$$f = ag - b\ b^1 \quad \text{and } h = bg + a\ b^1$$

The total primary current will be,

$$\underset{\cdot}{I}_p = \underset{\cdot}{I}_{oo} + \underset{\cdot}{I}_s$$

$$= i + j\ i_1 + f + j\ h$$

$$= i + f + j\ (i_1 + h)$$

$$= O + j\ K \quad O = (i + f) \quad K = (i_1 + h)$$

$$\underset{\cdot}{E}_o = E_1 + \underset{\cdot}{I}_p\ Z_p$$

$$= a + j\ b + (o + j\ k)\ (r_p + j\ x_p)$$

$$= L + j\ M$$

$$E_o = \sqrt{L^2 + M^2}$$

It will be shown later that the power input to a circuit taking $i - j\ i_1$ amperes at a potential $e + j\ e_1$ volts is $i\ e - i_1\ e_1$ watts or for a circuit taking $i - j\ i_1$ amperes a potential $e + j\ e_1$ volts, is $ie - i_1\ e_1$. Therefore it is evident that the power output of the transformer above is, P, and is the product of

$$e + j\ (\text{zero})$$

and $i - j\, i_1 = ei$ watts,

and the input will be $O + j\, k$ current and $L + j\, M$ volts or $(O\,L - K\,M)$ watts $= P_o$; hence the efficiency will be

$$\frac{P}{P_o} = \frac{\text{Output}}{\text{Input}} = \frac{ei}{O\,L - K\,M}$$

The power factor will be found by adding the two angles θ and θ_1 where θ = the angle between the primary current and the reference vector e and θ_1 = the angle between the impressed voltage and the vector represented by e.

Thus
$$\operatorname{Tan} \theta = \frac{K}{O}$$

and
$$\operatorname{Tan} \theta_1 = \frac{M}{L}$$

Power factor angle θ_3 may be found by adding θ and θ_1 when not in the same quadrant and subtracting when in the same quadrant.

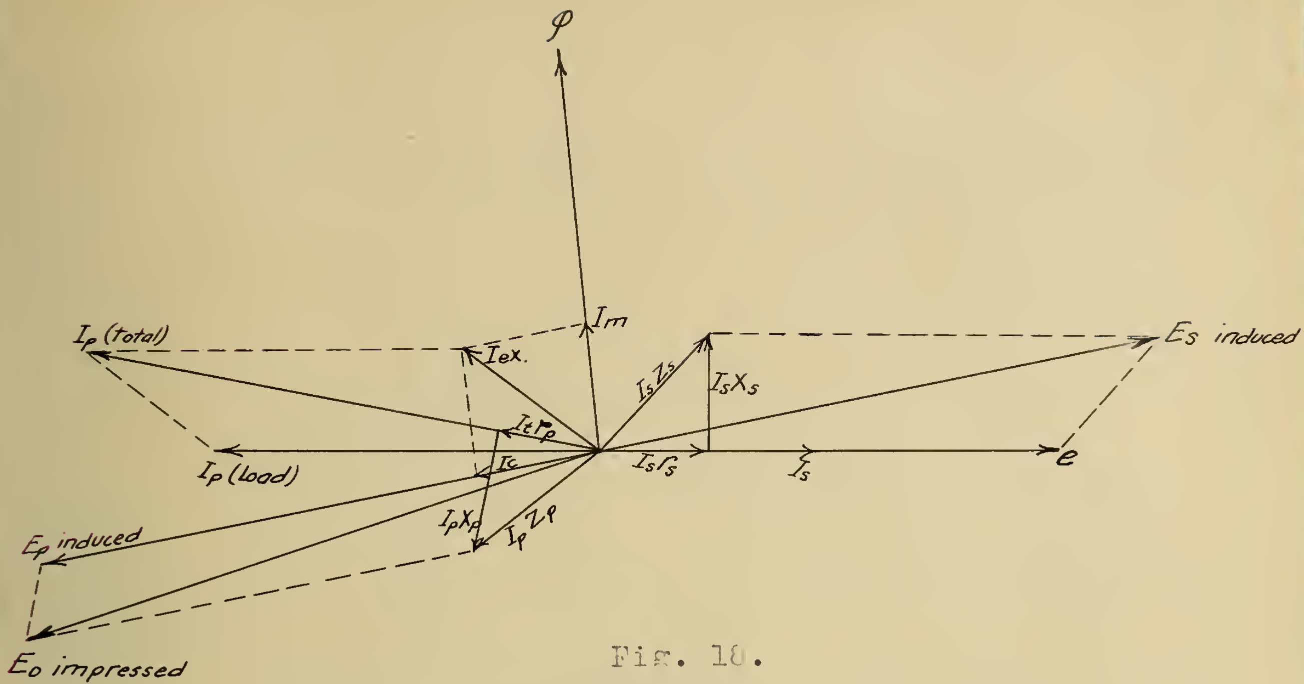

φ
Im
Ip (total)
Iex.
IsZs
IsXs
Es induced
Itrp
Ip (load)
Ic
Isrs
Is
e
IpXp
IpZp
Ep induced
Eo impressed

Fig. 18.

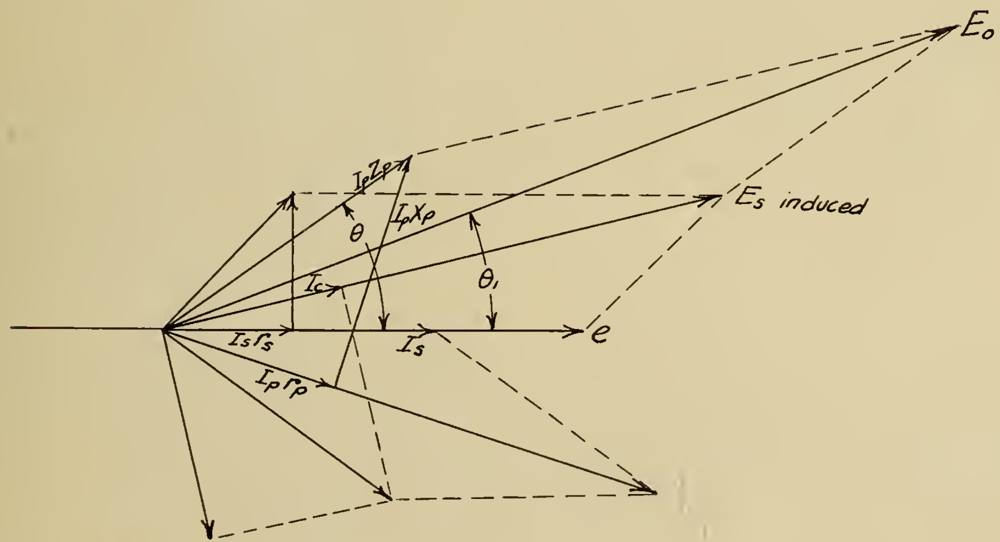

Eo
IpZp
IpXp
Es induced
θ
θ1
Ic
Isrs
Is
e
Iprp

Fig. 19.

Problem #13.

To prove Power = $ei - e_1 i_1$ for any condition

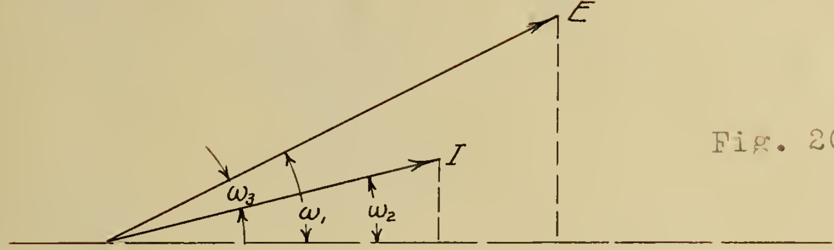


Fig. 20

Solution of Problem #13.

$$e = E \cos \omega_1 \qquad e_1 = E \sin \omega_1 \qquad E = E\,(\cos \omega_1 - j \sin \omega_1)$$

$$i = I \cos \omega_2 \qquad i_1 = I \sin \omega_2 \qquad I = I\,(\cos \omega_2 + j \sin \omega_2)$$

$$(\omega_1 - \omega_2) = \omega_3$$

Hence,

$ei + e_1 i_1$ should $= E\, I \cos \omega_3$ or

$$I\, E\,(\cos \omega_1 \cos \omega_2) + E\, I\,(\sin \omega_1 \sin \omega_2) = E\, I \cos \omega_3.$$

$$E\, I\,(\cos \omega_1 \cos \omega_2 + \sin \omega_1 \sin \omega_2) = E\, I \cos \omega_3.$$

Put,

$$\cos \omega_1 \cos \omega_2 + \sin \omega_1 \sin \omega_2 = \cos (\omega_1 - \omega_2)$$

$$= \cos \omega_3 .$$

Hence,

$$E\, I \cos \omega_3 = E\, I \cos \omega_3. \quad ei + e_1 i_1 = E\, I \cos \omega_3.$$

Problem 14.

A 10 K.W. transformer has constant secondary voltage.

Primary volts 2200

Secondary volts 110

Cycles 60

Test data: Open circuit, Current 3 amperes, secondary volts 110, watts 125, short circuit, full load current, volts 91.

Primary resistance = 5 ohms.

Secondary resistance = .01 ohm.

Find primary current, voltage and power factor with full load current in secondary, the secondary load to be (1), .8 P.F. lead, (2) P.F. unity, .8 P.F. lag.

Solution of Problem #14 by % method.

$$I_p = \frac{10000}{2200} = 4.55 \text{ amperes} = 1.00 \text{ in } \%.$$

$$I_s = \frac{10000}{110} = 91 \text{ amperes} = 1.00 \text{ in } \%.$$

$$r_p = 5 \text{ ohms}, \quad \frac{5 \times 4.55}{2200} = .01034 \text{ in } \%,$$

$$r_s = .01 \text{ ohm} \quad \frac{.01 \times 91}{110} = .00827$$

Impedance volts from test = 91 with full load current of 4.55 amperes.

$$\dot{Z} = \frac{E}{\dot{I}} = \frac{91}{4.55} = 20 \text{ ohms.} = \frac{91}{2200} = .04135 \%.$$

$\dot{Z} = r + jx$ where r = total resistance of the transformer and x = the total reactance of the transformer.

Total % resistance = .01034 + .00827 = .01861

$$\% \ \overline{\text{resistance}}^2 = \overline{.01861}^2 = .000346$$

$$\% \ \overline{\text{impedance}}^2 = \overline{.04135}^2 = .00171$$

$$.00171 - 000346 = .001364.$$

$\sqrt{.001364}$ = total reactance in % = .0369

$\% \, x_p = \% \, x_s = .01845$

From open circuit test.

$$I_c = \frac{125}{110} = 1.136 \qquad \% \, g = \frac{1.136}{91} = .012475$$

$$I_m = \sqrt{3^2 - \overline{1.136}^2} = 2.778 \qquad \% \, b^1 = \frac{2.778}{91} = .03053$$

At .8 power factor lead.

	$I = 4.55$	$i = 3.64$	$i_1 = 2.73$
% values	$L = 1.00$	$i = .8$	$i_1 = .60$

At unity power factor.

	$I = 4.55$	$i = 4.55$	$i_1 = 0$
% values	$L = 1.00$	$i = 1.00$	$i_1 = 0$

At .8 P. F. lag.

	$I = 4.55$	$i = 3.64$	$i_1 = 2.73$
% values	$I = 1.00$	$i = .8$	$i_1 = -.60$

The following Table II is found by substituting the above constants in the theory as derived on pages 27 to 30. The curves on page 36 show the variation of the various transformer characteristics when the power factor of the load is changed.

TABLE II

Power factor	Lead	Unity	Lag
	.80	1.00	.80
e	1.00	1.00	1.00
$i\,r_s$	.006616	.00827	.006616
$e + ir - i_1 x_s = a$	.9958	1.00827	1.0174
$i_1 x_s$	.0108	0	-.0108
$i_1 r_s$	.004963	0	-.004963
$i\,x_s$	.01476	.01845	.01476
$i_1 r_s + i\,x_s = b$	.01972	.01845	.00986
ag	.01248	.01257	.01270
$b\,b^1$	-.000602	-.000563	-.00031
$ag - b\,b^1 = f$	.013082	.013133	.01301
bg	.000246	.0002303	.0001228
$a\,b^1$	-.03054	-.03075	-.03108
$bg + a\,b^1 = h$	-.0305	-.03052	-.029957
$i_1 + h = K$	.59695	-.03052	-.629957
$i + f = 0$	.813082	1.01313	.81301
$o\,r$	.00842	.01047	.0084
$K\,x_p$	.01103	-.00563	-.01164
$a + {}^{P}o\,r_p - K\,x_p = L$	.9931	1.02437	1.03744
$o\,x_p$	.015	.01868	.0148
$K\,r_p$	.006165	-.003154	-.0065
$b + {}^{P}o\,x_p + K\,r_p = M$	.04088	.02398	.01816
$L^2 + M^2 = E_o$	.984	1.027	1.038
Output $P = e\,(i + j\,i^1)$	.8	1.00	.8
P_o input $= e_o i_o + e_o{}^1 i_o{}^1$	.8317	1.03727	.8240
Power factor	.825	.996	.783
Efficiency	.963	.9645	.959

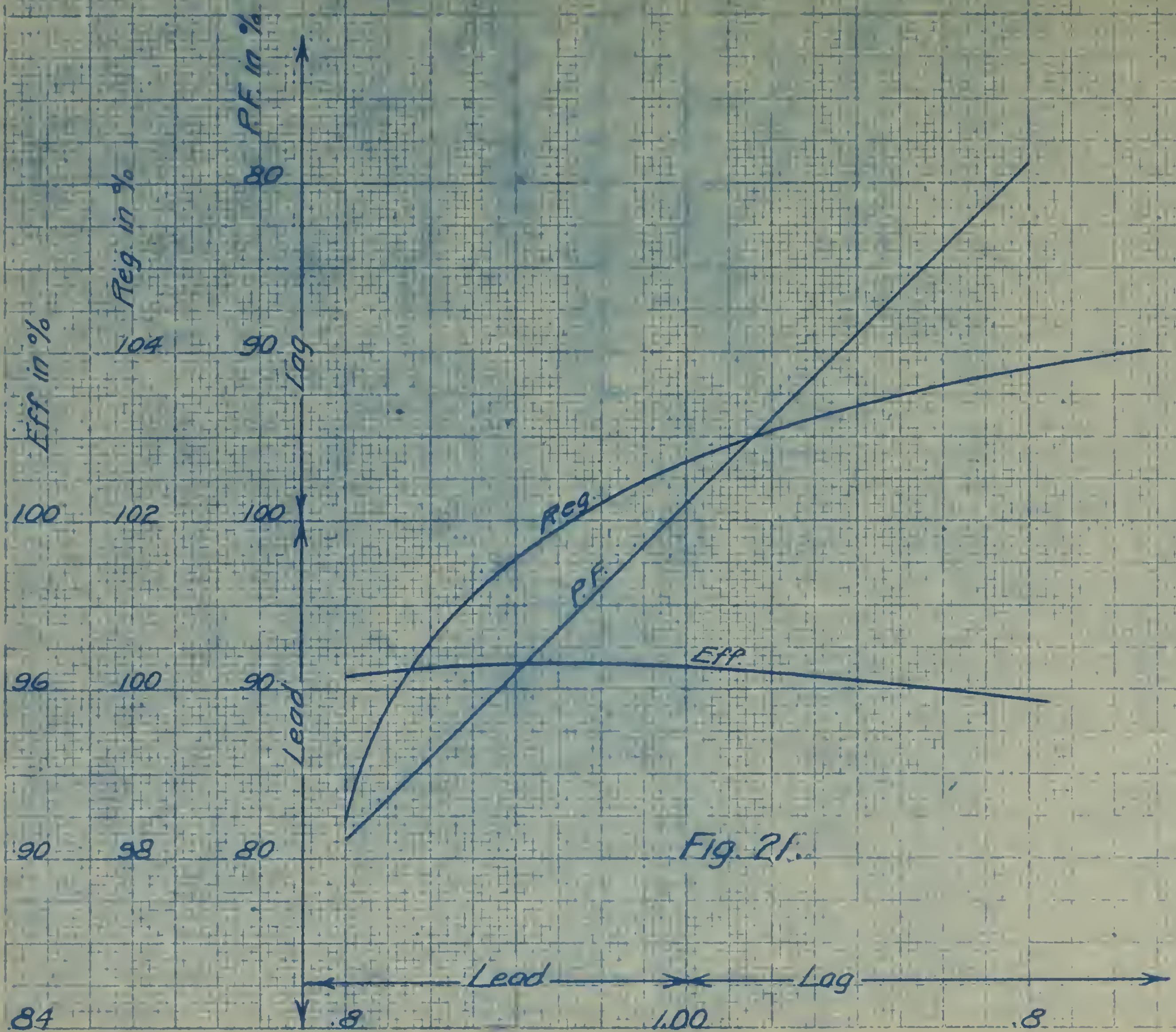


Fig. 21.

Curve Sheet 2

Problem #15.

Given $E_o = 1000 - 250\ j.$

$I_o = 10 - 20\ j.$

Find the power and power factor.

Problem #16.

Given 100 K.W. transformer with the following constants,

$E_s = 100$ $E_p = 2000$

$r_s = r_p = .01$ $x_s = x_p = .02$

Core loss = .02 $I_{mag} = .02$

Find the values of r_s, r_p, x_s, x_p in ohms. Find I_c I in amperes.

Problem # 17.

Given $r_p = r_s = .01$ $x_p = x_s = .02$

Core loss = .02 Magnetizing current = .05

Find the effect of regulation, power factor and efficiency.

1. Vary r_p and r_s from .01 to .30
2. Vary x_p and x_s from .01 to 30
3. Vary core loss from .01 to .20
4. Vary magnetizing current from .02 to .50.

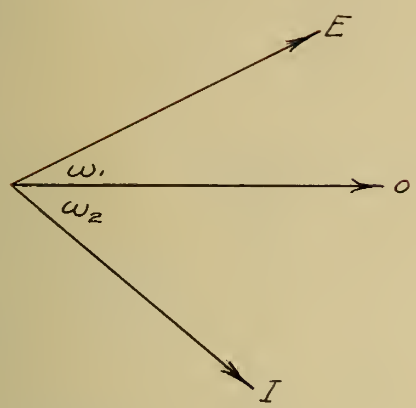
All values above are in % and are equivalent values. Calculate a full load and unity power factor.

Solution of Problem #15.

Power $= e\ i - e_1\ i_1$

$= 1000 \times 10 - 250 \times 20$

= 10000 - 5000 = 5000 watts.



$$\text{Tan}\ \omega_1 = \frac{250}{1000} = .25 \qquad \omega_1 = 14^\circ\ 2'$$

$$\text{Tan}\ \omega_2 = \frac{-20}{10} = -2 \qquad \omega_2 = -63^\circ\ 26'$$

$$\omega_1 + \omega_2 = \omega_3 \qquad \omega_3 = 70\ \ 28'$$

$$\text{P. F.} = \cos \omega_3 = \cos 77^\circ\ 28' = .2170$$

Solution of Problem #16.

$$\frac{100000}{100} = 1000 = I_s$$

$$\frac{100000}{100} = 50 = I_p$$

$1000 \times r_s = .01 \times 100$	$r_s = .001$ ohms
$50 \times r_p = .01 \times 2000$	$r_p = .4$ ohms
$1000 \times x_s = .02 \times 100$	$x_s = .002$ ohms
$50 \times x_p = .02 \times 2000$	$x_p = .8$ ohms

$I_c = .02 \times 1000 = 20$ amp. on low side

$I_c = .02 \times 50 = 1$ amp. on high side

$I_m = .05 \times 1000 = 50$ amp. on low side

$I_m = .05 \times 50 = 2\ 1/2$ amp. on high side.

Problem #17 was solved by substituting the constants in the general equations derived on pages [illegible] and [illegible]. The curves accompanying the various tables are self explanatory.

TABLE III

Effect of Variation of Resistance on Transformer Operation.

Unity Power Factor $i = 1.0$ $i_1 = 0$

P.F. of load	1.0	1.0	1.0	1.0	1.0
e	1.0	1.0	1.0	1.0	1.0
$r_s = r_p$	.01	.04	.08	.16	.32
$i\ r_s$	.01	.04	.08	.16	.32
$i_1 x_s$	0	0	0	0	0
$e + ir_s - i_1 x_s = a$	1.01	1.04	1.08	1.16	1.32
$i_1 r_s$	0	0	0	0	0
$i\ x_s$	.02	.02	.02	.02	.02
$i_1 r_s + i\ x_s = b$	.02	.02	.02	.02	.02
g	.02	.02	.02	.02	.02
ag	.0202	.0208	.0216	.0232	.0264
b^1	-.05	-.05	-.05	-.05	-.05
$b\ b^1$	-.001	-.001	-.001	-.001	-.001
$ag - b\ b^1 = f$	.0212	.0218	.0226	.0242	.0274
bg	.0004	.0004	.0004	.0004	.0004
ab^1	-.0505	-.052	-.054	-.058	-.066
$bg + ab^1 = h$	-.0501	-.0516	-.0536	-.0576	-.0656
$i_1 - h = k$	-.0501	-.0516	-.0536	-.0576	-.0656
$i + f = o$	1.0212	1.0218	1.0226	1.0242	1.0274
or_p	.10212	.040872	.081808	.16387	.328768
kx_p	-.001002	-.001032	-.001072	-.001152	.001312
$a + or_p - kx_p = L$	1.021214	1.081904	1.16288	1.32502	1.65008
ox_p	.020424	.020436	.020452	.020484	.020548
kr_p	-.000501	-.000516	-.000536	-.000576	-.000656
$b + ox_p + kr_p = M$	.039923	.03992	.039916	.039908	.039802
$L^2 + M^2 = E_o$	1.022	1.084	1.164	1.325	1.645
Reg. =	.022	.084	.164	.325	.645
$P = e(i + j_1 i_1)$	1.0	1.0	1.0	1.0	1.0
$P_o = e_o i_o + e_o i_o$	1.04286	1.10343	1.18702	1.35577	1.69265
$\text{Tan}\ \theta_1 = \frac{k}{o}$	-.049	-.0505	-.0524	-.0562	-.0638
$\text{Tan}\ \theta_2 = \frac{M}{L}$	.0391	.0369	.0343	.0301	.02416
$\theta_3 =$	$5^{\circ}2'$	5°	$4^{\circ}58'$	$4^{\circ}57'$	$4^{\circ}56'$
$\text{Eff} = \frac{P}{P_o}$	.959	.917	.854	.738	.591
$\text{Cos}\ \theta_3 = \text{P.F.}$	.99617	.99616	.99625	.99627	.99630

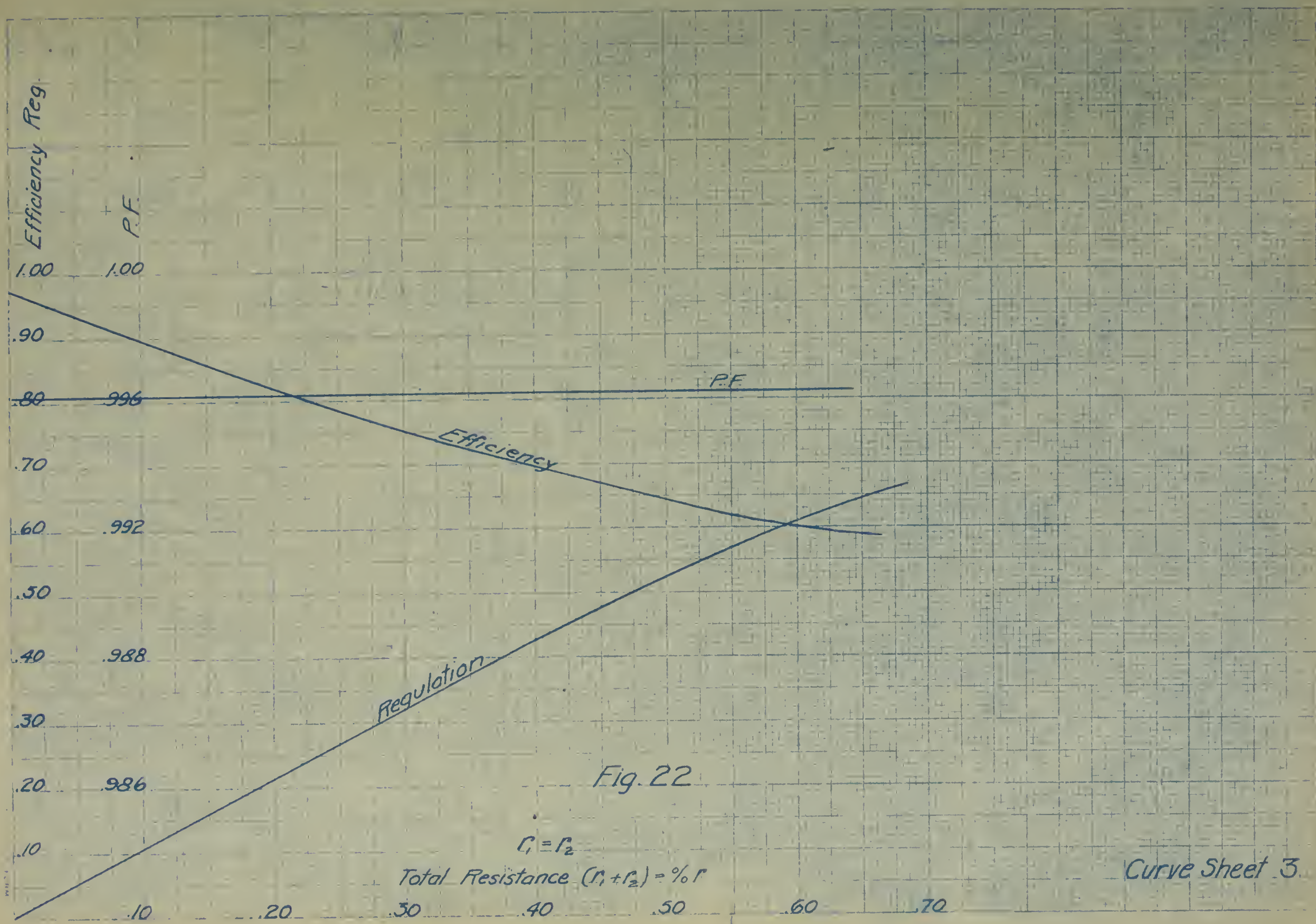


Fig. 22

Curve Sheet 3

TABLE IV

Effect of Variation of Reactance on Transformer Operation.

Unity Power Factor $i = 1.0$ $i_1 = 0$

P.F. of load	1.0	1.0	1.0	1.0	1.0
e	1.0	1.0	1.0	1.0	1.0
$i\ r_s$	.01	.01	.01	.01	.01
$i_1 x_s$	0	0	0	0	0
$e + i\ r_s - i_1 x_s = a$	1.01	1.01	1.01	1.01	1.01
$i_1 r_s$	0	0	0	0	0
$x_s = x_p$	.02	.04	.08	.16	.32
$i\ x_s$	.02	.04	.08	.16	.32
$i_1 r_s + i\ x_s = b$	.02	.04	.08	.16	.32
g	.02	.02	.02	.02	.02
ag	.0202	.0202	.0202	.0202	.0202
b^1	-.05	-.05	-.05	-.05	-.05
$b\ b^1$	-.001	-.002	-.004	-.008	-.016
$ag - b\ b^1 = f$	.0212	.0222	.0242	.0282	.0362
bg	.0004	.0008	.0016	.0032	.0064
$a\ b^1$	-.0505	-.0505	-.0505	-.0505	-.0505
$bg - a\ b^1 = h$	-.0501	-.0497	-.0489	-.0473	-.0441
$i_1 - h = k$	-.0501	-.0497	-.0489	-.0473	-.0441
$i - f = 0$	1.0212	1.0222	1.0242	1.0282	1.0362
or_p	.010212	.010222	.010242	.010282	.010362
kx_p	-.001002	-.001988	-.003912	-.007568	-.014112
$a + or_p - kx_p = L$	1.021214	1.022210	1.024154	1.02765	1.034474
ox_p	.020424	.040888	.081936	.164512	.331584
kr_p	-.000501	-.000497	-.000489	-.000473	-.000441
$b + ox_p + kr_p = M$	.039923	.080391	.161449	.324039	.651143
$L^2 + M^2 = E_o$	1.022	1.025	1.032	1.078	1.221
Ref. =	.022	.025	.032	.078	.221
$P = e(i + j\ i_1)$	1.0	1.0	1.0	1.0	1.0
$P = e_o i_o + e_o^1 i_o^1$	1.04084	1.0409	1.04104	1.04145	1.04323
Tan θ_1	-.0491	-.0486	-.04775	-.046	-.02457
Tan θ_2	.039	.0536	.1576	.305	.6285
θ_3	$5^\circ 3'$	$6^\circ 0'$	$11^\circ 42'$	$19^\circ 36'$	$34^\circ 35'$
Cos θ_3 = P.F.	.996	.9945	.9792	.942	.8233
Eff.	.961	.9605	.9601	.9595	.9584

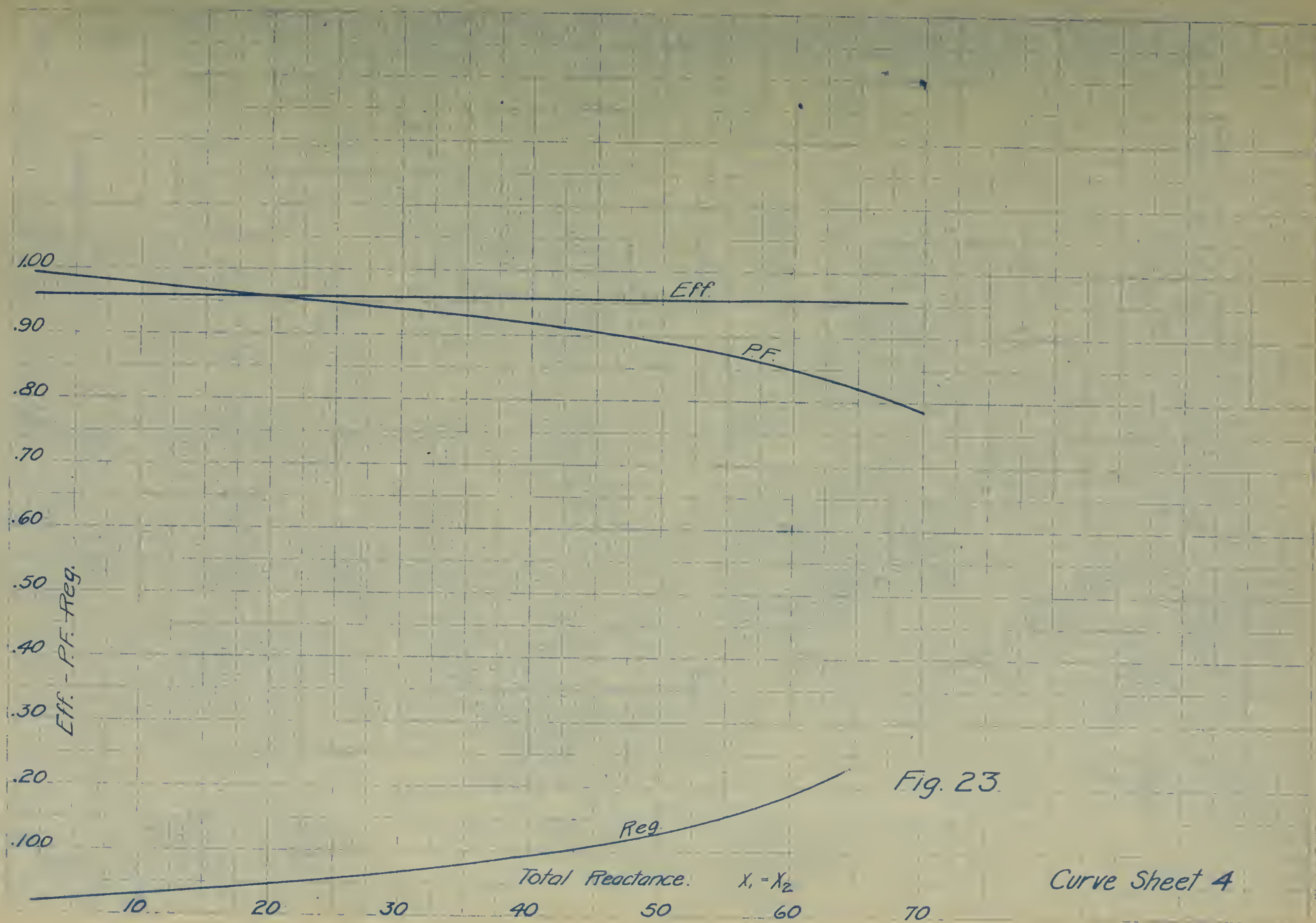


Fig. 23

Curve Sheet 4

TABLE V

Effect of Variation of Core Loss on the Operation of a Transformer.

Unity Power Factor $i = 1.0$ $i_1 = 0$

P.F. of load	1.0	1.0	1.0	1.0	1.0
e	1.0	1.0	1.0	1.0	1.0
$i\ r_s$	.01	.01	.01	.01	.01
$i_1 x_s$	0	0	0	0	0
$e + ir_s - i_1 x_s = a$	1.01	1.01	1.01	1.01	1.01
$i_1 r_s$	0	0	0	0	0
ix_s	.02	.02	.02	.02	.02
$i_1 r_s + i\ x_s = b$	.02	.02	.02	.02	.02
g	.02	.04	.08	.16	.32
ag	.0202	.0404	.0808	.1616	.3232
b^1	-.05	-.05	-.05	-.05	-.05
$b\ b^1$	-.001	-.001	-.001	-.001	-.001
$ag - b\ b^1 = f$	.0212	.0412	.0812	.1626	.3242
bg	.0004	.0008	.0016	.0032	.0064
ab^1	-.0505	-.0505	-.0505	-.0505	-.0505
$bg + ab^1 = h$	-.0501	-.0497	-.0489	-.0473	-.0441
$i_1 + h = k$	-.0501	-.0497	-.0489	-.0473	-.0441
$i - f = 0$	1.0212	1.0412	1.0812	1.1626	1.3242
or_p	.010212	.010412	.010812	.011626	.013242
kx_p	-.001002	-.000994	-.000978	-.000946	-.000882
$a + or_p - kx_p = L$	1.021312	1.02141	1.02179	1.02257	1.02412
ox_p	.020424	.020824	.021624	.023252	.026484
kr_p	-.000501	-.000497	-.000489	-.000473	-.000441
$b + ox_p + kr_p = M$	.03992	.04032	.04113	.04277	.04604
$L^2 + M^2 = E_o$	1.022	1.0225	1.023	1.023	1.0244
$I =$	1.0	1.0	1.0	1.0	1.0
$I_o =$	1.0405	1.061	1.108	1.185	1.352
Tan θ_1	-.0491	-.0477	-.0455	-.0407	-.0332
Tan θ_2	.0391	.0395	.04115	.0418	.0449
θ_3	5°3'	5°0'	4°57'	4°43'	4°29'
Cos θ_3	.99612	.99619	.99627	.9966	.99694
Reg.	.022	.0225	.023	.0235	.0244
Eff.	.961	.9425	.902	.844	.751

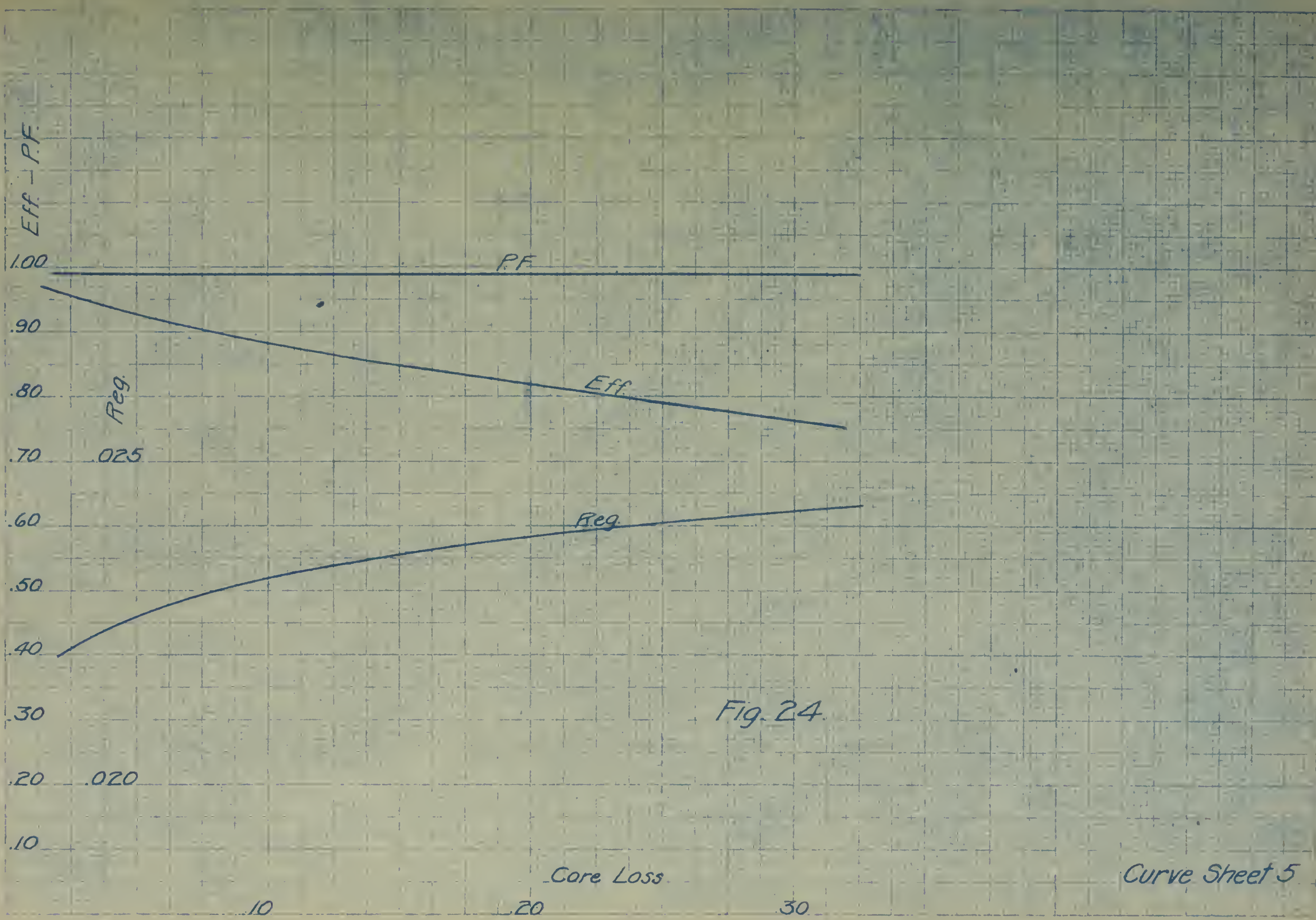


Fig. 24.

Curve Sheet 5

TABLE VI

Effect of Variation of Magnetizing Current on the Operation of Transformers

Unity Power Factor $i = 10$ $i_1 = 0$

P.F.	1.0	1.0	1.0	1.0	1.0
e	1.0	1.0	1.0	1.0	1.0
ir_s	.01	.01	.01	.01	.01
i_1x_s	0	0	0	0	0
$e + ir_s - i_1x_s = a$	1.01	1.01	1.01	1.01	1.01
i_1r_s	0	0	0	0	0
$i\ x_s$	.02	.02	.02	.02	.02
$i_1r_s - ix_s = b$	.02	.02	.02	.02	.02
g	.02	.02	.02	.02	.02
ag	.0202	.0202	.0202	.0202	.0202
b^1	-.05	-.10	-.20	-.40	-.60
$b\ b^1$	-.001	-.002	-.004	-.008	-.012
$ag - b\ b^1 = f$	.0212	.0222	.0242	.0282	.0322
bg	.0004	.0004	.0004	.0004	.0004
$a\ b^1$	-.0505	-.1010	-.2020	-.4040	-.6060
$bg - ab^1 = h$	-.0501	-.1006	-.2016	-.4036	-.6056
$i_1 + h = k$	-.0501	-.1006	-.2016	-.4036	-.6056
$i + f = 0$	1.0212	1.0222	1.0242	1.0282	1.0322
$a + or_p - kx_p = L$	1.021214	1.022234	1.02427	1.028554	1.033422
ox_p	.020424	.020444	.020484	.020564	.020644
kr_p	-.000501	-.001006	-.002016	-.004036	-.006056
$b + ox_p + kr_p = M$	.039925	.039458	.038468	.036528	.034588
$E_o = L^2 + M^2$	1.022	1.0225	1.0245	1.029	1.034
Reg.	.022	.0225	.0245	.029	.034
Output	1.0	1.0	1.0	1.0	1.0
Input	1.041	1.04103	1.0411	1.0435	1.0471
Efficiency	.961	.9605	.9601	.9582	.9555
Tan θ_1	.0491	.0984	.1967	.3923	.587
Tan θ_2	.0391	.0386	.03755	.0345	.03244
θ_3	$5^\circ 5'$	$7^\circ 50'$	$13^\circ 17'$	$23^\circ 23'$	$32^\circ 16'$
Cos θ_3 = P.F.	.99612	.99067	.9732	.91787	.8455

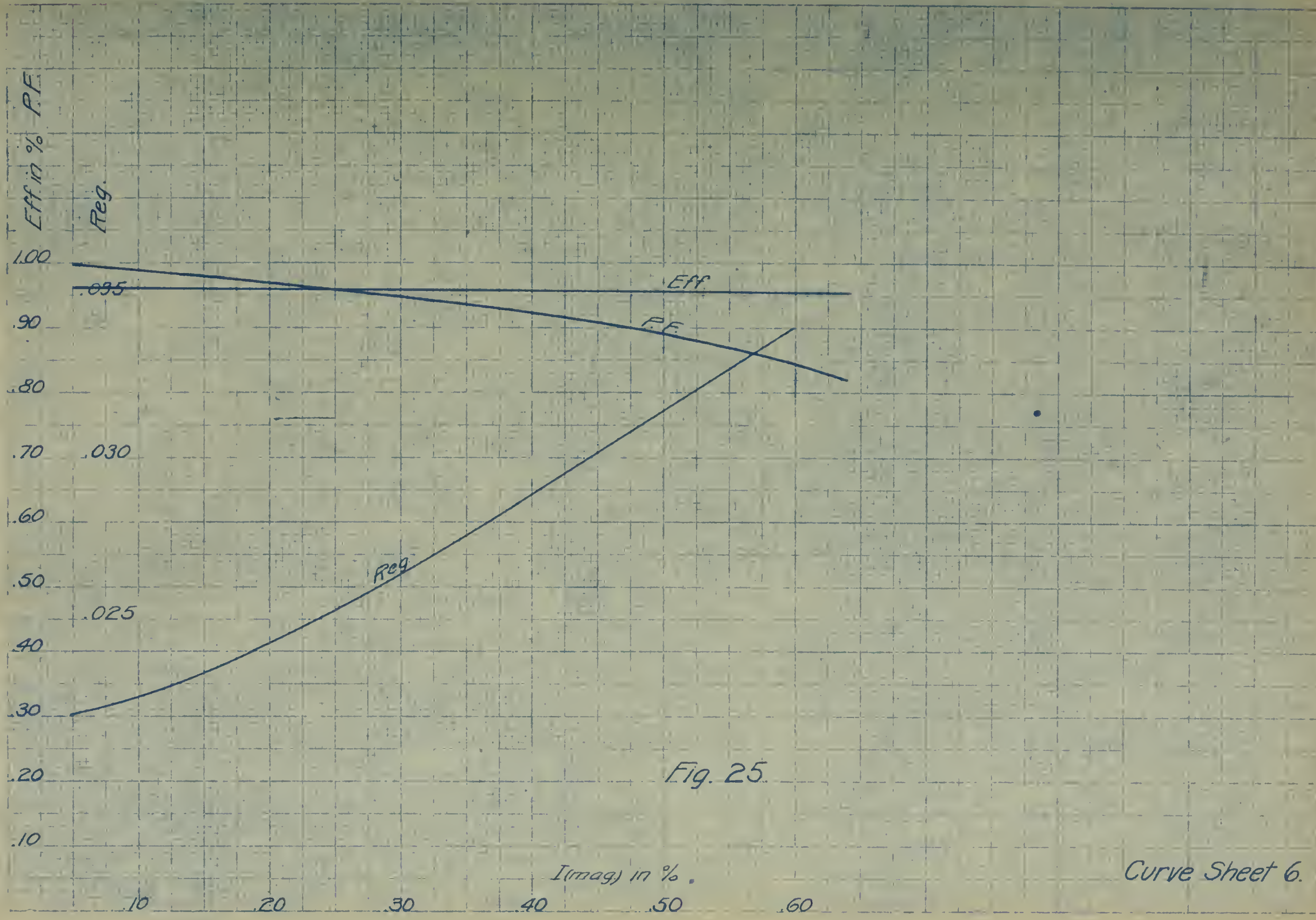


Fig. 25.

Curve Sheet 6.

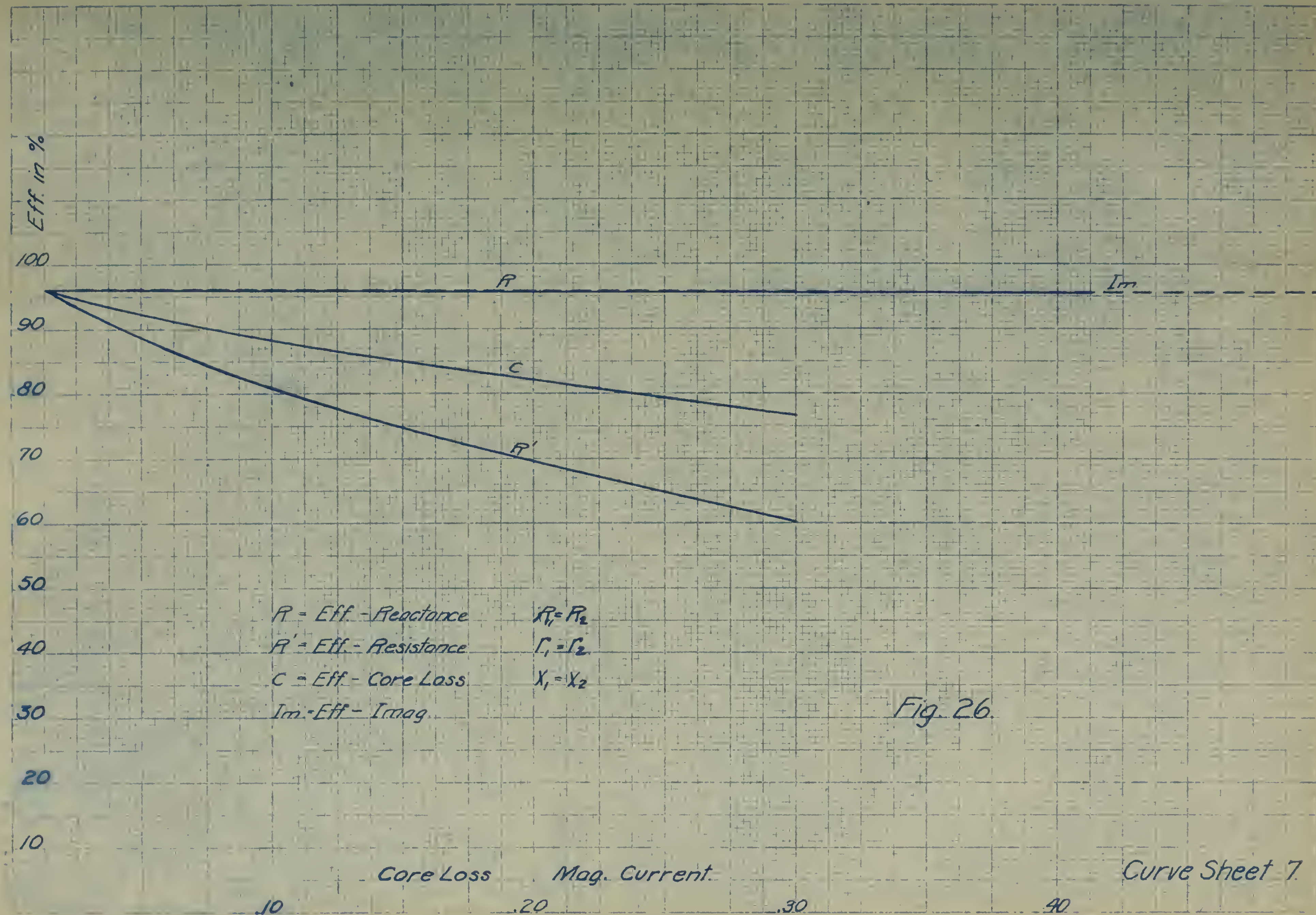


Fig. 26.

Curve Sheet 7

Problem #18.

A transformer has the following constants,

$r_1 = r_2 = .01 \quad x_1 = x_2 = .02$ Core loss = .02

$I_{mag} = .05$

If the voltage is increased 25% and the frequency kept constant what will be the output with the same total losses? What will be the % values of r_1, r_2, x_1, x_2, g and b based on the new rating. What will be the rating and the constants when the voltage is decreased 25%. Use iron curve as given on page 54 and make the hysteresis loss = .80 of iron loss and eddy current loss = .20% of the iron loss. Frequency to remain constant in all cases.

Solution of Problem #18.

Increase E 25%, then E = 1.25 and the density must be 1.25.

Original hysteresis loss $= K\beta^{1.6} = .016 = .80$ total iron loss.

Original eddy loss $= K_1\beta^{2} = .004 = .20$ total iron loss.

With E = 1.25 the new hysteresis loss $= \overline{1.25\beta}^{1.6} = 1.43$

With E = 1.25 the new eddy loss $= \overline{1.25\beta}^{2} = 1.562$.

New losses as a % of the old = 1.43 x .016 = .02288.

New losses as a % of the old = 1.562 x .004 = .00625.

Total losses as a % of the old = .02913

Since the sum of the iron and I^2R losses cannot be greater than .04 it is evident that the I^2R loss must be,

$$.04 - .02913 = .01087 = I^2R \quad \text{and}$$

if .02 = 100% I^2R = loss in the original case .01087 = 54.35% = loss in the new case and the current will be the square root of the losses, or

$$I = \sqrt{.5435} = .737 \text{ of the original current}$$

and the power output will be,

$$.737 \times 1.25 = .922 \text{ of the original rating.}$$

The I R drop will be $\frac{.737 \times .02}{1.25} = .01178$,

and since $R_s = R_p$ we have,

$$R_s = R_p = .00589 = \% \text{ resistance new rating.}$$

The IX drop will be $\frac{.737 \times .04}{1.25} = .02356$.

Since $X_s = X_p$ we have $X_s = X_p = .01178 = \%$ reactance new rating.

The new core loss based on the new rating will be,

$$\frac{.02913}{.922} = .0316 = g \text{ expressed in \% of new rating.}$$

Old density = 8000 x 6.45 = 51400 lines per sq. in.

New density = 1.25 x 8000 x 6.45 = 61500 lines per sq. in.

Old m.m.f. required = 6.

New m.m.f. required = 9.

% increase in m.m.f. based on old current rating will be $\frac{9}{6} = 1.50$.

% m.m.f. of old rating will be 1.50 x .05 = .075.

% m.m.f. based on the new current rating will be $\frac{.075}{.737} = .1018 = \%$.

Decrease E 25% or E = .75 and hence the density will be .75 the original density. With E = .75 the new hysteresis loss will be,

$$\overline{.75\beta}^{1.6} = .631 \text{ the original loss and the new eddy}$$

loss will be $\overline{.75\beta}^{2} = .562$ the original loss.

Therefore $.631 \times .016 =$ .01 1= % hys. of old rating

$.562 \times .004 =$ .00224 % eddy of old rating

.01234 % total of old rating

Hence there will be left for I^2R loss

$.04 - .0123 = .0277$ as a % of original I^2R.

Hence if .02 = 100% original loss,

.0277 = 138.5% original loss which may be consumed by I^2R under the new conditions.

New current rating as a % of the old current rating will be

$\sqrt{1.385} = 1.176$ and the new power rating will be

E I = $1.176 \times .75 = .883$ = the new power rating.

With 1.176 as the new current the I R drop will be $\frac{1.176 \times .02}{.75} =$ .0314 and since $R_s = R_p$ we have $R_s = R_p = \frac{.0314}{2} = .0157$ = % reactance based on the new rating. The reactance will be $\frac{1.176 \times .04}{.75}$ = .0628 and since $X_s = X_p$ it is evident that

$X_s = X_p = .0314$ expressed in % of new rating. The new core loss in % will be,

$\frac{.01234}{.883} = .01397$ = % g (new rating.)

Original density = $8000 \times 6.45 = 51400$ lines per sq. in.

New density = $.75 \times 8000 \times 6.45 = 38500$ lines per sq. in.

Original m.m.f. required = 6.

New M.M.F. required = 4.5

% decrease in m.m.f. base on old current rating is $\frac{4.5}{6.0} = .75$

% m.m.f. of old rating will be $.75 \times .05 = .0375$

% m.m.f. based on new rating will be,

$\frac{.0375}{1.176} = .0319$ = %.

TABLE VII

Summation of Constants.

E	.75	1.00	1.25
I	1.176	1.00	.757
$\beta r_p = r_s$	.0157	.01	.00589
$\beta x_p = x_s$	.0314	.02	.01178
βg	.01397	.02	.0316
b	.0319	.05	.1018

Problem #19A.

A coil, shown in Fig. 27, has 180 turns of #10 copper wire. What is its inductance?

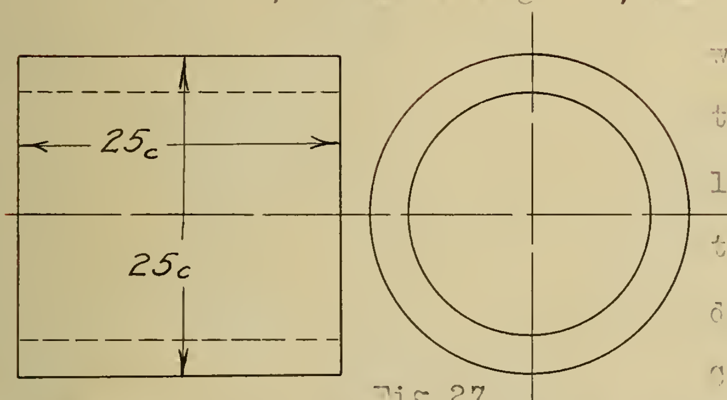


Fig.27.

19B. What is the resistance of the coil at 25 degrees C? at 75 degrees C?

19C. What will be the flux passing through the coil with 25 amperes D.C. flowing? With 25 amperes A.C.?

19D. What is the reactance of the coil at 60 cycles?

19E. What is the e.m.f. drop across the coil with 25 amperes flowing, taken from direct current mains?

19F. What is the e.m.f. drop across the coil with 25 amperes flowing, taken from 60 cycle A.C. mains? T = 75 degrees C.

19G. What is the power factor of the circuit in 19F?

19H. What flux will be required to induce an e.m.f. of 220 volts in the coil Fig. 27, if connected to a 60 cycle circuit?

19I. What magnetizing current will be required to produce this flux from 60 cycle mains?

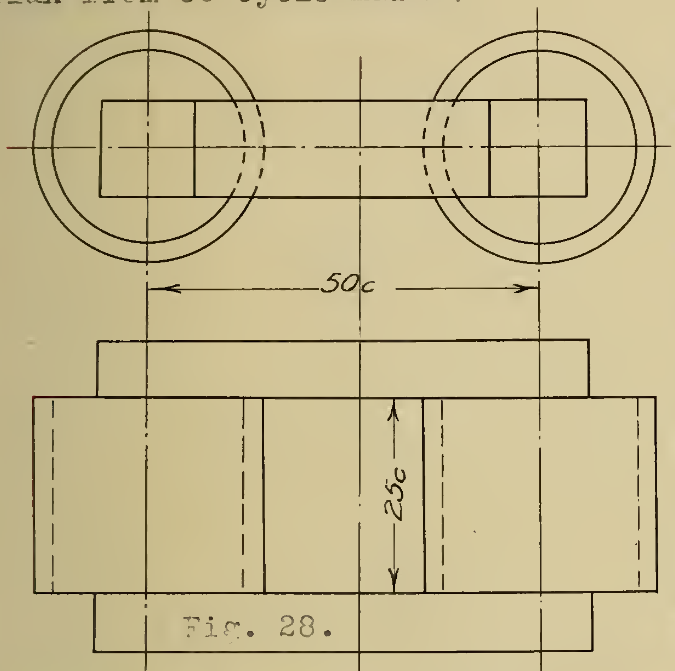


Fig. 28.

19J. An iron core the area of which is one-fifth the area of coil, square in cross section and its length of magnetic circuit as shown in Fig. 28 is placed inside the coil. Suppost the steel to be of quality shown by curve for armature stampings, P. 64, Thomalen. What will be the value of the magnetizing currents, for same flux as in 19E, considering all that flux as passing through the iron?

19K. With the magnetizing current flowing as in 19J, what will be the number of lines in the coil outside of the iron? What will be the density in the iron? What e.m.f. will be induced in each coil if each has 180 turns #10 wire?

19L. What flux will pass through the iron core with 25 amperes magnetizing current at 60 cycles flowing? How many lines through the coil outside the iron?

19M. What e.m.f. will be induced in coil #1? What e.m.f. will be induced in coil #2 having the same number of turns? Flux the same as in 19L.

19N. The core is now cut at A and B and an air gap of 1 cm. equally divided between the two places, is introduced. What magnetizing current would be required in coil #1 so that the flux shall induce 220 volts in coil #2 at 60 cycles?

Solution of #19A.

Diameter of #10 B. & S. gauge wire = .107 inches.

Wind coil in 2 layers of 90 turns each.

Inside diameter of coil will be,

25 - (4 x .107 x 2.54) = 23.913 cms.

Inside area of coil = 448 sq. cms.

Assume length of mangetic path = length of coil = 25 cms.

L may be defined in two ways, (i.e.)

$$L = \frac{n\varphi}{i}$$ or L = the number of interlinkages per unit current and again,

$$L\frac{di}{dt} = e = N\frac{d\varphi}{dt}$$

hence if

$$e = N\frac{d\varphi}{dt} = L\frac{di}{dt}$$ then, $\frac{d\varphi}{dt} = L\,N\,\frac{di}{dt}$, but $\beta = \mu H$ and

$$H = \frac{.4\Pi N i}{L}$$ where i is in practical units of current,

again,

$$a\beta = \varphi = \mu a H \quad \text{and}$$

$$\varphi = \mu a \frac{.4\Pi N i}{L}$$

$$e = N\frac{d\varphi}{dt} = \mu\frac{a\ .4\ \Pi\ N^2}{L}\ \frac{di}{dt} = L\frac{di}{dt}\ \mu\frac{a\ .4\ \Pi\ N^2}{L}\ \frac{di}{dt}$$

Or $$L = \mu\frac{a\ .4\ \Pi\ N^2}{L}$$

L in problem above = $\frac{1 \times 448 \times .4 \times \Pi \times 180 \times 180}{25}$

Curve Sheet 8

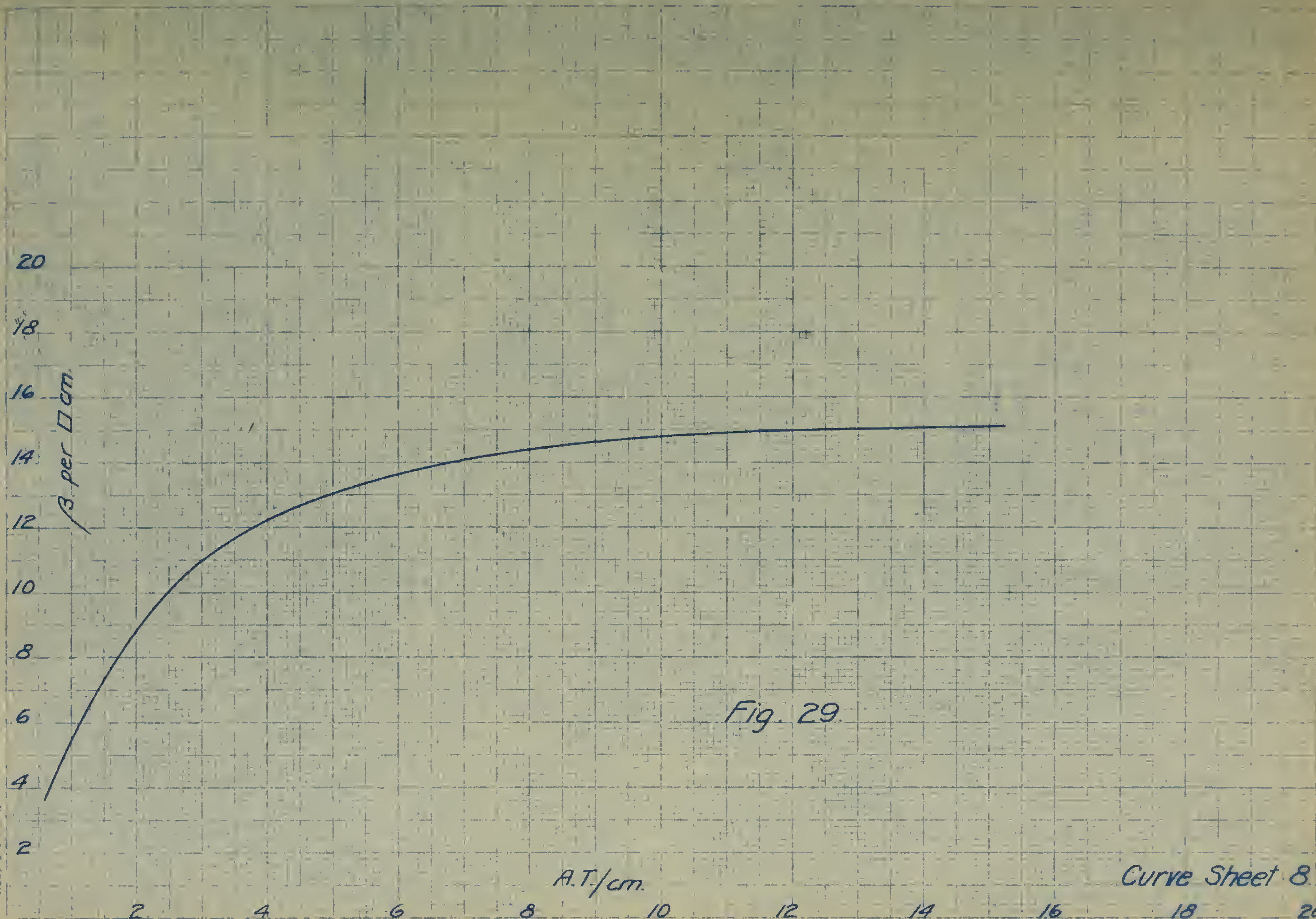


Fig. 29.

$$L = 729000 \text{ cms.} = \frac{729000}{10^8} = .00729 \text{ henrys.}$$

Problem 19B.

$$F^o = 9/5\ C^o + 32^o \qquad C^o = 5/9\ (F^o - 32^o).$$

$$75^o\ F = 5/9\ (75^o - 32^o) = 23.9^o\ C.$$

Let R_1 = resistance at temperature T_1

R_2 = resistance at temperature T_2

R_o = resistance at temperature 0^oC

$\triangle$ = temperature coefficient.

$$R_1 = R_o\ (1 + \triangle\ T_1) \qquad R_2 = R_o\ (1 + \triangle\ T_2)$$

$$R_2 - R_1 = R_o\ (T_2 - T_1)\triangle$$

$$\frac{R_2 - R_1}{R_o} = (T_2 - T_1)\triangle$$

In most practical cases T_1 seldom greater than 15^o to 25^o C, hence $R_1 = R_o$ makes a very slight error and we may write

$$\frac{R_2 - R_1}{R_1} = .004\ (T \quad - T \ ) \text{ or } \frac{R_2 - R_1}{R_1} = .4\ (T \quad - T \ )$$

which corresponds to a change in resistance of .4% for each degree C change in temperature. Resistance of #10 B. & S. wire at 25^o C, if we assume its substance is 1 ohm at 23.9^o C, will be

$$\frac{R_2 - 1.0}{1.0} = .004\ (25 - 23.9) = 1.0044 \text{ ohms per } 1000'.$$

Resistance at 75^o will be,

$$\frac{R_2 - 1.0}{1.0} = .004\ (75 - 23.9) = 1.2044 \text{ ohms per } 1000'.$$

Mean diameter of coil = 24.457 cms.

Length of wire per coil = $\pi \times 24.457 \times 180 = 13930$ cms. $\frac{13830}{12 \times 2.54}$

= 454 ft.

Resistance of coil at 25^o C = .454 x 1.0044 = .456 ohms.

Resistance of coil at 75° C = .454 x 1.2044 = .547 ohms.

Problem #19C.

$$\beta = \mu H = \frac{.4\pi Ni}{L} = \frac{.4\pi\ 180 \times 25}{25} = 226$$

φ_{max} = 226 x 448 = 101300 with 25 amperes D.C.

$\varphi_{max} = \sqrt{2} \times 101{,}303$ = 142.800 with 25 amperes A.C.

Problem #19D.

x = 2πfL

= 2πf x .00729 = 377 x .00729 = 2.748 ohms.

Problem #19E.

Resistance at 75° C is .547 ohms

Drop with 25 amperes D.C. = 25 x .547 = 13.675 volts.

Drop with 25 amperes D.C. at 25° C = 25 x .456 = 11.5 volts.

Problem #19F.

Using 75° C as the temperature.

$$Z = \sqrt{2.748^2 + .547^2} = 2.8 \text{ ohms}$$

I Z = 25 x 2.8 = 70 volts.

Problem #19G.

Power factor = $\cos \omega = \frac{r}{z}$ = .1953.

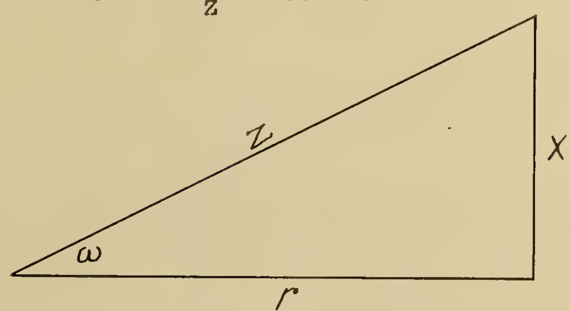


Fig. 30.

Problem #19H.

Since $E = 4.44 \times f \times \varphi \times N \times 10^{-8}$

$$\varphi = \frac{E \times 10^8}{4.44 \times f\ N} = \frac{220 \times 10^8}{4.44 \times 60 \times 180} = 463.000$$

Problem #19I.

The density in the coil will be,

$$= \frac{\varphi}{\text{area}} = \frac{465.000}{448} = 1033 \text{ lines per sq. cm.}$$

Since $\mu = 1$ for air $= \frac{.4\pi NI}{L}$ hence,

$$1033 = \frac{.4\pi\ 180\ I_{max}}{25} \qquad I_{max} = 114.25$$

$$I_{eff} = \frac{114.25}{\sqrt{2}} = 81 \text{ amperes.}$$

Problem #19J.

Area of iron core $= \frac{1}{5}$ area of coil $= --- = 89.6$ sq. cm.

Cross section = 9.46 cms. x 9.46 cms.

Density in core will be $\frac{465.000}{89.6} = 5170$ lines per sq. cm.

From curve the ampere turns necessary per cm. at this density = .90.

Length of path = 2 x 50 x 2 x 25 = 150 cms. + 18.92

Total ampere turns will be 168.92 x .90 = 152

$$\frac{152}{180} = .844 \text{ amp.} \quad (\text{max.})$$

$$\frac{.844}{2} = .599 \text{ amperes} \quad (\text{eff.})$$

Problem #19K

An area of air space in the coil will be 448 - 89.6 = 358.4 sq. cm. =

$$\frac{.4\ N\ i_{max.}\pi}{L} = \frac{.4 \times 180 \times .844\pi}{25} = 7.64 \text{ lines per sq. cm.}$$

$$7.64 \times 358.4 = 2753 = \varphi.$$

Density in iron will be the same as before or 5170 lines per sq.cm.

E.M.F. induced in the secondary will be,

$$E_s = 4.44 \times 60 \times 463.000 \times 180 \times 10^{8} = 221 \text{ volts.}$$

If we assume the flux in air is in phase with the flux in the iron then the total flux cutting the secondary will be 468170 X and the induced primary voltage (neglecting the resistance) will be,

$$E_s = 4.44 \times 60 \times 468170 \times 180 \times 10^{-8} = 225.8$$

Problem #19L.

$$\text{Total A. T.}_{max} = 25\sqrt{2} \times 180 = 5350.$$

Length of magnetic path = 168.92 cms.

$$\text{A.T. per cm.} = \frac{5320}{168.92} = 31.65$$

From curve for 3.165 A. T. per cm. = about 15300 lines per sq. cm.

$$\varphi \text{ in iron} = 15300 \times 89.6 - 1,370,000$$

$$\varphi \text{ in air} = \frac{.4\pi \times 180 \times 25 \times \sqrt{2} \times 358.4}{25} = 114.400$$

Problem #19N.

$$220 = \pi \times 60 \times 180 \times 10^{-8} \times 4.44 = 548.000$$

Assume density in gap is same as the density in air. A.T. for gap will be, $\frac{.4\pi Ni}{L} = \frac{458.000}{89.6} = 5120.$

$$i = \frac{5120 \times 1}{.4\pi \times 180} = 22.6 \text{ amp. (max.)}$$

A.T. for iron will be from curve corresponding to density of 5120 lines per sq. cm. .9 per cm.

Length of path = 167.92

Total A.T. = 167.92 x .9 = 151

$$I_{max} = \frac{151}{180} = .84$$

Total I_{max} = .84 - 22.6 = 23.44 $\quad I_{eff} = \frac{23.44}{2} = 16.62$ amp.

IV

CONSTANT CURRENT TRANSFORMER

The constant current transformer is so designed that, when constant potential is impressed on the primary winding, constant current will be delivered by the secondary regardless of load or power factor of the load. The constant current transformer is often confused with the constant current regulator although they are designed along very different lines, the essential difference being that the constant current transformer may have very large difference between the primary or impressed voltage and the secondary induced or load voltage, while the constant current regulator acts only as a variable reactance in series with the load. The following figures represent in a general way the two types of machines discussed above.

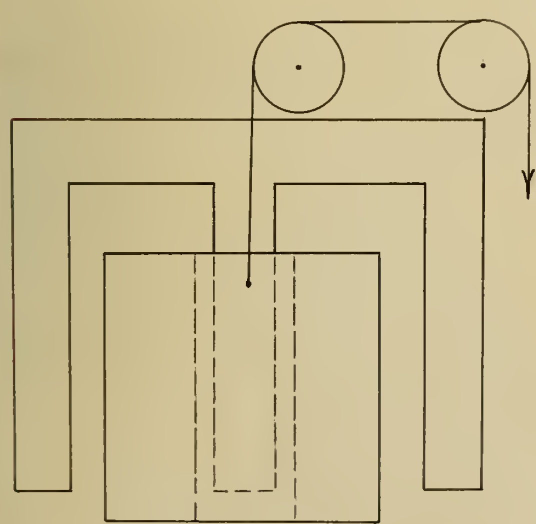


Fig. 31.

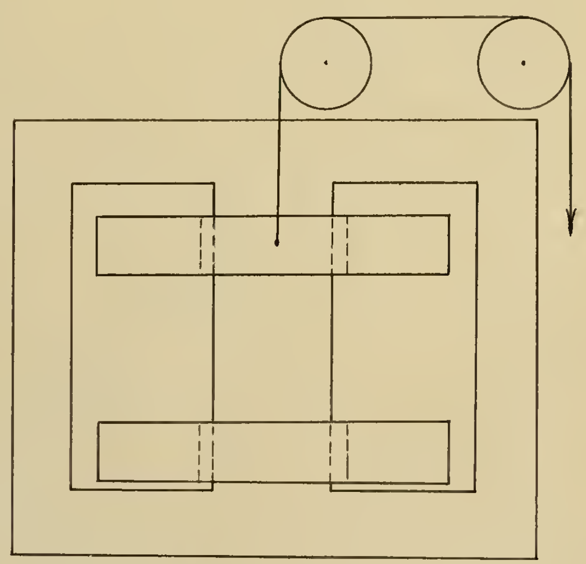


Fig. 32.

Figure 31 represents the current regulator which acts as follows: When the resistance of the load decreases the current in the line will increase and thereby cause a greater force of attraction between the coil and the iron core, hence the coil which is nearly balanced by the weight W will move in a vertical direction which forces the iron core deeper into the coil and thus increases the reactance of the coil. Now if the coil is properly balanced by the counter weight it will take a very slight increase in current to maintain this new position although the reactance of the coil in the new position may be very much increased. Thus it is evident that the current remains practically constant, the surplus terminal e.m.f. being consumed by an increase in reactance. From the following vector diagram it is seen that the power factor of the total circuit, when regulated for constant current by a regulator, may be very good at full load and very poor at light loads.

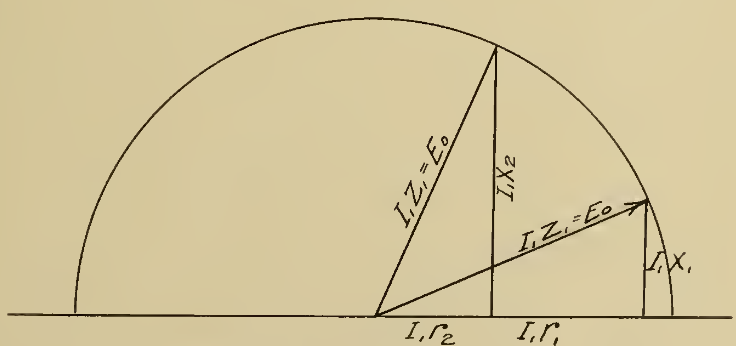


Fig. 33.

The diagram also shows that at full load the value of r is large as compared to x while at light loads the value of x is very large in comparison to r. It is also evident that such regulation is obtained through a large range, only at the sacrifice of power

With the constant current transformer conditions are some what different. First the primary voltage need not be impressed on the circuit, but may be steped up or down any desired amount before connecting to the load. Therefore for a given circuit where the voltage available is much greater than the voltage of the constant current circuit it is possible to maintain constant current at a much higher power factor. This is perhaps the greatest advantage of the constant current transformer over the current regulator.

The vector diagram for the constant current transformer for any given load and power factor is identical with the vector diagram of a potential transformer having the same constants, although it must be borne in mind that the reactance will vary for each condition of loading and hence a new diagram would be necessary each time the load is changed. The constant current transformer and constant current regulator are used almost exclusively for series street lights with either arc or tungsten lamps.

A transformer built as shown below is sometimes used to regulate for constant current. This transformer does not operate on the principle of variable reactance but produces the same effect as would be caused by connecting a large reactance in series with a resistance load, that is the reactance is large in comparison to the resistance. By proper adjustment this arrangement will give a variation of current of about 2% when the resistance is changed 15%. This scheme of course means a circuit of low power factor.

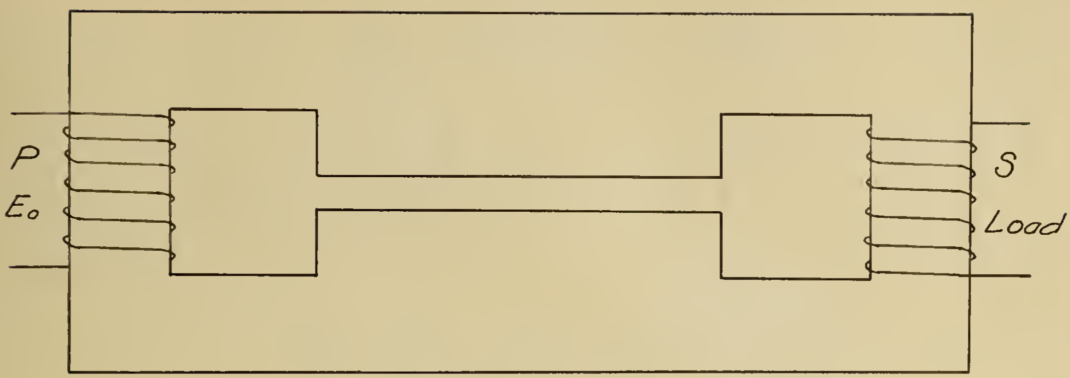


Fig. 34.

The following problem will demonstrate to a limited extent the action of the constant current transformer.

Rating 4 K.W. 440 volts primary, 6.6 amperes secondary.

Resistance of primary .67 ohms.

Resistance of secondary 1.25 ohms.

Test data. Open circuit (approximate values.)

$E = 440$ $I = .50$ Core loss 100 watts.

Short circuit test $E = 440$ $I_p = 8.6$ $I_s = 6.2$

Short circuit test $E = 109$ $I_p = 2$, coils apart (max.)

Short circuit test $E = 102$ $I_p = 10$, coils together.

(a) Find the limits of the reactance.

(b) Calculate the full load efficiency assuming the coils to be maximum distance apart at full load.

Solution (a).

From short circuit test, $Z = \frac{109}{2} = 54.5$ ohms.

Ratio of transformation is $\frac{6.2}{8.6} = .721$.

$R_p + R_s \left(\frac{6.2}{8.6}\right)^2 = .67 + .65 = 1.32$ = total equiv. = R resistance.

Total reactance $= X = \sqrt{Z^2 - R^2}$ and in the position of maximum reactance $X = \sqrt{\overline{54.4}^2 - \overline{1.32}^2} = 54.55$ ohms.

For minimum reactance,

$$Z = \frac{102}{10} = 10.2 \text{ ohms.}$$

$$X = \sqrt{\overline{10.2}^2 - \overline{1.32}^2} = 10.09 \text{ ohms.}$$

(b) Efficiency at full load.

At full load consider the coils are the maximum distance apart and that $I_s = 6.6$ amperes.

Then,

$$I_p = \frac{I_s}{.721} = \frac{6.6}{.721} = 9.15 \text{ amp.}$$

$$I_p^2 R = \overline{9.15}^2 \times .67 = 56.1$$

$$I_s^p R = \overline{6.6}^2 \times 1.25 = 54.5$$

Total $I^2R = 110.6$ watts.

Core loss = 100 watts.

Total loss = 210.6 watts.

Output = 4000 watts.

Output + losses = 4210.6 watts.

$$\text{Eff.} = \frac{4000}{4210.6} = 94.9\% \text{ efficiency.}$$

Forces existing between the coils of a constant current transformer

(a) Force repulsion or attraction?

Current in same direction Fig. 35 (a). Current in opposite direction Fig. 35 (b).

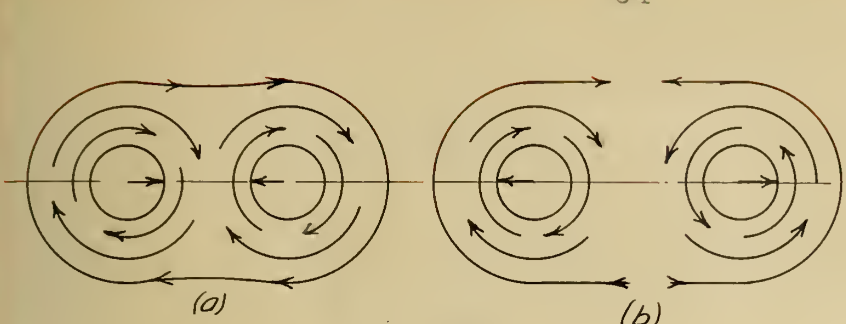


Fig. 35.

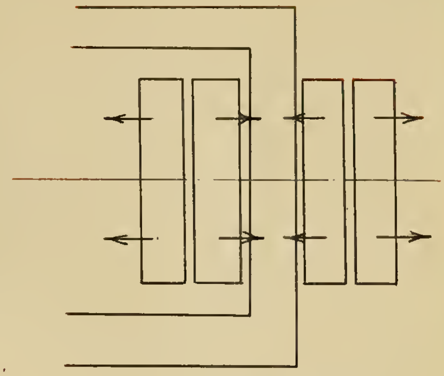


Fig. 36.

Let L = the inductance of the transformer.

i = the current flowing.

Thus the energy stored in the magnetic field will be $\frac{1}{2} L i^2$; neglecting the energy necessary to keep current flowing which will be zero provided the current remains constant in strength.

(a) Suppose the two coils are exactly together and the current is kept constant, the leakage will be zero. Next suppose the current is kept constant and the coils are pulled apart the distance L' as indicated. The average force necessary to do this is F. Work done = FL' and hence $FL' = \frac{1}{2} L i^2$.

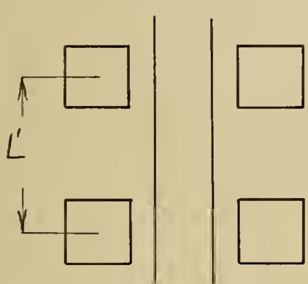


Fig. 36a.

F in grams L' in cms.

FL' = gram cm.

$FL' \times 980 \times 10^{-7}$ = Joules = J.

L in henrys i in amperes.

$\frac{1}{2} L i^2 J = FL' \quad 980 \times 10^{-7}$

$X = 2\pi f L$ on short circuit $= \frac{E}{Z}$

R is small $i = \frac{E}{Z} = \frac{E}{X}$

$L' F = \frac{1}{2} \frac{x}{2\pi f} \frac{E^2}{x^2} = FL'$

$$F = \frac{E^2}{4\pi f \times 980 \times 10^{-7}}$$

or

$$F = \frac{L\ i^2}{2L} = \frac{L\ i\ E\ 10^7}{2L\ \ x\ 980} = \frac{L\ E\ i\ 10^7}{2L\ 2\pi f L\ 980}$$

$$F_{grams} = \frac{810\ e\ i}{f\ L'} \quad \checkmark$$

The force existing between the coils of a transformer.

(a) When two parallel conductors are carrying current in the same direction a force exists due to the magnetic field which tends to force the conductors together as illustrated in the following figure.

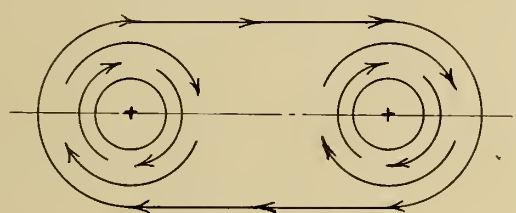


Fig. 37.

When the current flows in the opposite direction the force tends to drive the conductors apart as is illustrated below.

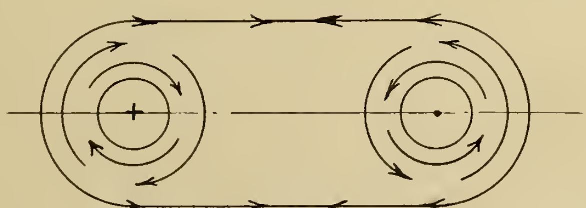


Fig. 38.

Now take the case of the actual transformer as illustrated below.

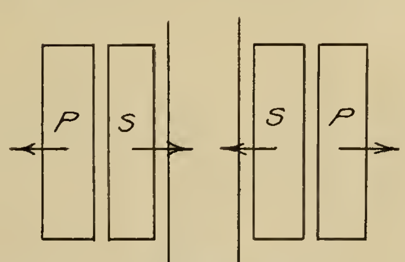


Fig. 39.

It is evident from previous discussions that the current in the primary flows in a direction opposite to that of the secondary and hence if any lines pass between the coils they must produce a force tending to push the coils apart as indicated above.

Let L = the inductance of the transformer.

i = the current flowing.

Then the energy stored in the magnetic field as has been previously shows in

$$E_n = \frac{1}{2} L i^2 .$$

Neglecting the resistance of the coils the energy necessary to keep the current flowing will be zero provided the current remains constant in strength. Now suppose that the two coils are occupying the same space and the current is maintained constant. The leakage flux under this condition must be zero. Again suppose the current is kept constant and the coils are pulled apart the distance r as indicated below.

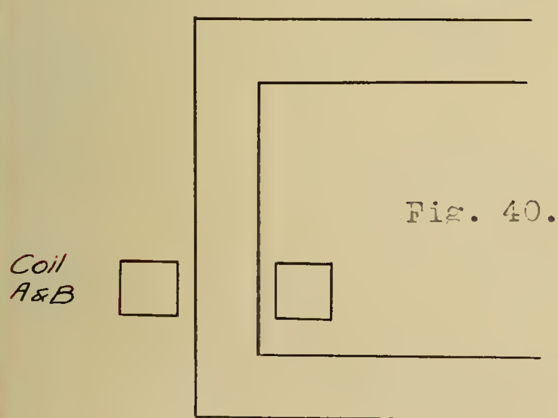


Fig. 40.

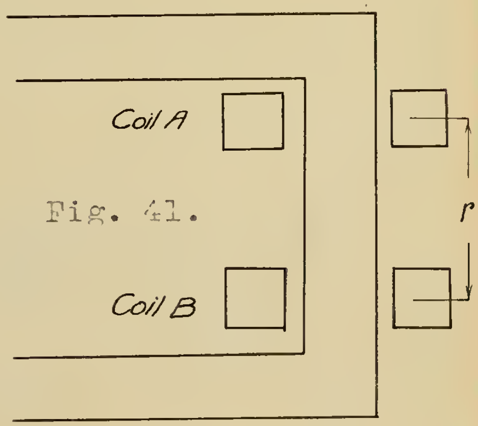


Fig. 41.

Let F = the average force necessary to pull the coils apart. Then the work done is Fr and hence $Fr = \frac{1}{2} L i^2$.

Let F = force in grams and

r = distance in cms. then

Fr = gram cms. and

$$Fr \times 980 \times 10^{-7} = \text{Joules} = J.$$

L is in Henrys. i in amperes.

$$J = \frac{1}{2} L\, i = Fr \quad 980 \times 10^{-7}$$

$X = 2\pi fL$ and for a transformer on chort circuit $i = \frac{E}{Z} = \frac{E}{X}$.

$$Fr = \frac{1}{2}\ \frac{x}{2\pi f}\ \frac{E^2}{X^2}$$

$$F = \frac{E^2}{4\pi\ f \times 980 \times 10^{-7}}$$

$$F = \frac{Li^2}{2\ r} = \frac{Li\ E\ 10^7}{2r \times 980} = \frac{L\ E\ i\ 10^7}{2r\ 2\pi f\ 980}$$

$$F \text{ in grams} = \frac{810\ E\ i}{f\ r}$$

In the commercial potential transformer there is no way to measure the force between coils but with the constant current transformer the conditions are such that this force can be determined. The following data was taken on a constant current transformer and will aid in showing the accuracy of this method of calculation although no great accuracy is claimed for the data.

$E = 110 \qquad I = 6.9 \qquad L = 7.6$ cms. $\qquad f = 60$

Force as measured = 53 ounces = 3.3 pounds Calculated:

$$F = \frac{810 \times 110 \times 6.9}{60 \times 7.6} = 1350 \text{ grams.}$$

$$\frac{1350}{453} = 2.98 \text{ pounds Calculated force.}$$

$$\frac{2.98}{3.3} \times 100 = 90\%$$ or an error of 10%, which is

very good for rough calculations since the data was not taken with an idea of obtaining very accurate results.

Take case of short circuit on this transformer with normal impressed voltage and the coils very close together.

Since the resistance will be very small it is evident that X = Z on short circuit, and from previous calculations Z = 10 and

$$I = \frac{E}{Z} = \frac{440}{10} = 44 \text{ amperes.}$$

This is about 7 times normal current and we would expect the force to be about 7 times as great as under normal conditions, however, this is not true since r is very small, and it is found that, since the primary current is 7 times normal and hence the secondary current is 7 times normal, that the force is about 49 times the normal exerted force, or the force varies about as the square of the primary or secondary current.

Consider the case of a constant potential transformer of 1000 K.W. capacity, 25 cycles.

$$e = 10{,}000 \text{ volts.}$$

$$x = .04$$

$$r = .005$$

$$z = .04$$

$$\text{Current on short circuit} = \frac{10{,}000}{Z} = \frac{1.00}{.04} = 25 \text{ times full load}$$

or

$$25 \times \frac{1{,}000{,}000}{10{,}000} = 2500 \text{ amperes.}$$

Distance between coils 10 cms. = r.

$$F = \frac{810\ e\ i}{fr} = \frac{810 \times 10{,}000 \times 2500}{25 \times 10}$$

$$= 81{,}000{,}000 \text{ grams.}$$

$$F = 178,000 \text{ pounds.}$$

$$F = 89 \text{ tons.}$$

Then if the current should rise very rapidly to 2500 amperes on short circuit the transformer coils would receive a blow equal to 89 tons. Unprotected transformers are often torn to pieces by this action.

V

CURRENT TRANSFORMER

The current transformer is used, as its name implies, to transform large currents to small currents or vise versa. The object being to reduce the current carrying capacities of measuring instruments and to insulate these instruments from the main circuit.

The current transformer has a primary and secondary coil, a magnetic circuit formed of iron and is insulated very much as any other transformer.

The diagram representing its connection in the line is as shown in figure 42.

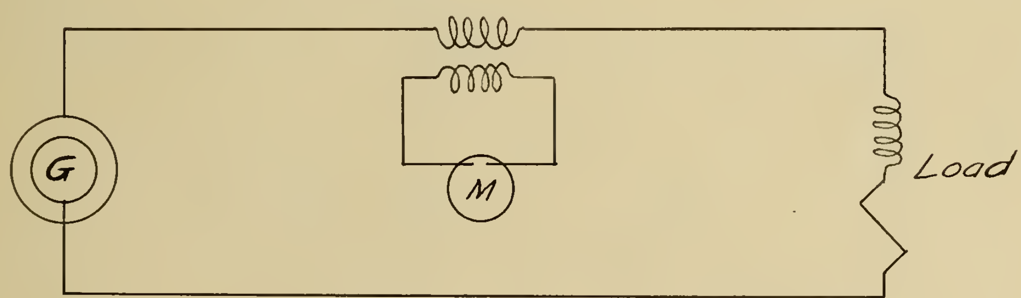


Fig. 42.

From the above it is evident that the equivalent electric circuit will be as shown in figure 43.

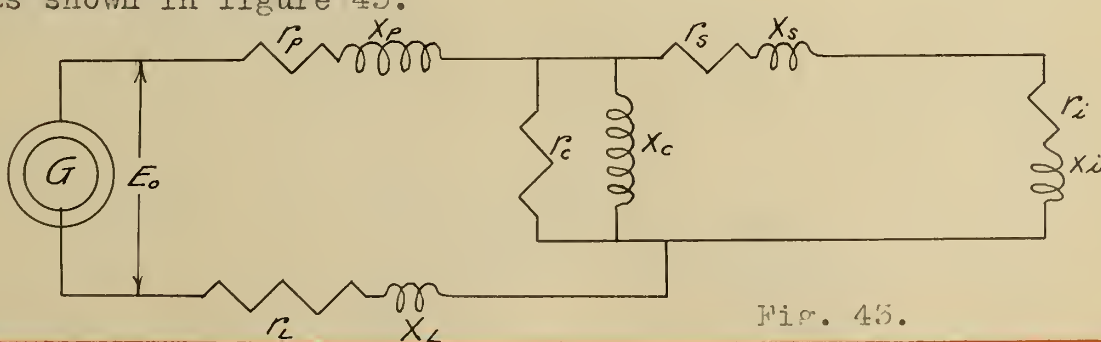


Fig. 43.

Where,

$r_p + j\, x_p$ = impedance of the primary.

$r_s + j\, x_s$ = impedance of the secondary.

r_c represents the core loss.

x_c represents the magnetizing current.

$r_i + j\, x_i$ = impedance of the instrument.

$r_L + j\, X_L$ = impedance of the load.

E_o = generator voltage.

Approximate values for a current transformer may be taken as,

$r_p = r_s = .001$ in %	$x_i = 0$ in %
$x_p = x_s = .005$ in %	$r_i = .005$ in %
$r_c = 100$ in %	$x_c = 1000$ in %

Connections of current transformer.

(a) Primary in series with load.

(b) Secondary short circuited through instrument.

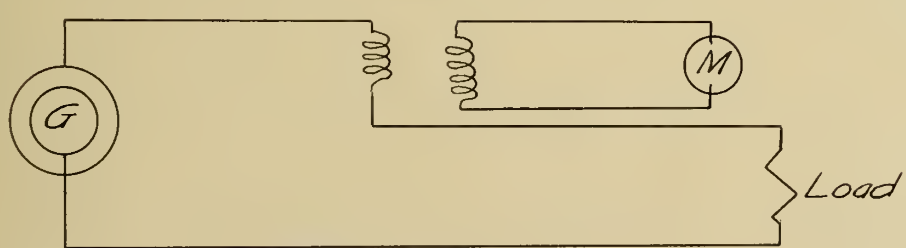


Fig. 44.

It must be remembered that secondary resistance and reactance must be reduced to equivalent primary values.

There are at least two conditions which should be investigated,

(a) Ratio of I_p to I_s for varying load current and varying power factor of load.

(b) Phase difference between I_s and I_p for varying loads and varying load power factor.

The first is important when either current or watts are to be measured. While the second is important where watts are to be measured through the transformer.

Reasonable current transformer constants are about as follows:

$r_p = .001 \quad x_p = .005 \quad r_c = 100 \quad r_{meter} = .005$

$r_s = .001 \quad x_s = .005 \quad x_c = 1000 \quad x_{meter} = 0$

Some calculated data using the above transformer constants give the following,

P. F. of Load = 1.00

Load	R	I_L	I_{meter}	$\measuredangle I_L$	$\measuredangle I$	Diff.
Full	1	.993	.993	34'	37'	3'
1/2	2	.497	.495	17'	19'	2'
1/4	4	.249	.249	8'	5'	3'

The above table shows the current transformer to be very accurate at unity power factor.

Increase of meter resistance, (as leads for instance)

$r = .01$

Full load $\quad I_L = .988 \quad I = .987$

1/4 load $\quad I_L = .249 \quad I = .2488$

Which shows that if the resistance is increased from .005 to .01 that the ratio is still constant.

Varying P. F. of Load.

P.F. load	.75 lag	1.0	.75 lead
R_L	1.0	1.0	1.0

I_L	(633 - j .447)	.993	(.645 + j .474)
I_m	(.6339 - j .447)	.9915	(.644 + j .4731)
∢ I load	37° 2'	34'	36° 59'
∢ I meter	36° 59'	37'	36° 59'
Phase diff.	3'	3'	0

Variation of P.F. at full load makes very little error in readings of transformer.

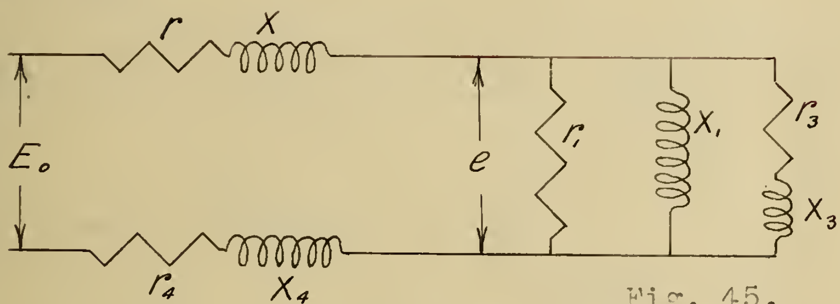


Fig. 45.

$$I_1 = \frac{e}{r_1 + Jx_1} = \frac{e\,(r_1 - J\,x_1)}{r^2 + x^2} = e\,(g_1 + J\,b_1) = e\,Y_1$$

$$g_1 = \frac{r_1}{r_1^2 + x_1^2} \qquad b_1 = \frac{-x_1}{r_1^2 + x_1^2}$$

$$I_2 = e\,Y_2 \qquad g_2 = \frac{r_2}{r_2^2 + x_2^2} \qquad b_2 = \frac{-x_2}{r_2^2 + x_2^2}$$

$$I_3 = e\,Y_3 \qquad g_3 = \frac{r_3}{r_3^2 + x_3^2} \qquad b_3 = \frac{-x_3}{r_3^2 + x_3^2}$$

I_o = total current.

$$I_o = I_1 + I_2 + I_3 = e\,(Y_1 + Y_2 + Y_3)$$

$$= e\,(g_1 + g_2 + g_3) + J\,(b_1 + b_2 + b_3)$$

The vector diagram of the above conditions is given in figure 46 where I_o is the total current flowing in all three branches.

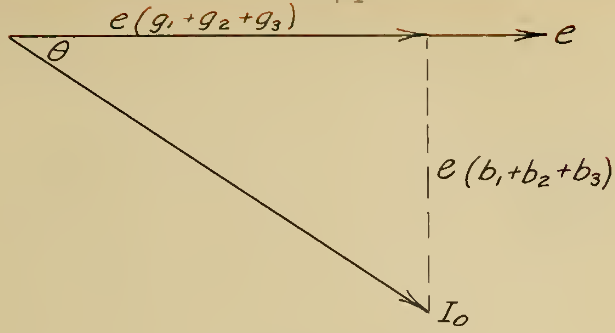


Fig. 46.

Hence the power factor of the total current with respect to e is given by $\tan\theta = \frac{b_1 + b_2 + b_3}{g_1 + g_2 + g_3}$.

The power factor of the current flowing through the ammeter is represented by the relation of I_3 to e, and is illustrated in fig.47.

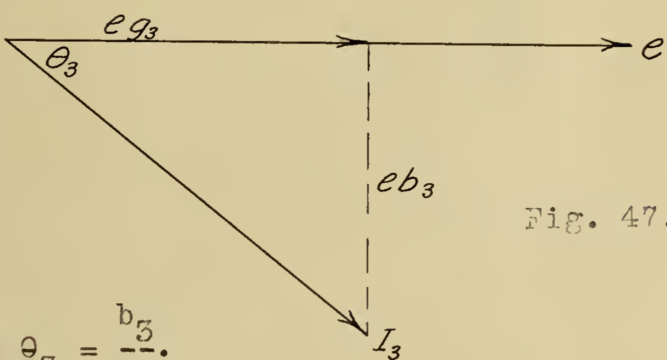


Fig. 47.

$$\text{Tan } \theta_3 = \frac{b_3}{g_3}.$$

From the above it is evident that I_o is divided into two components at right angles to each other or,

$$I_o = i + j\, i_1 \quad \text{where}$$

$$i + j\, i_1 = e \quad (g_1 + g_2 + g_3) + J\,(b_1 + b_2 + b_3)$$

$$i = e\,(g_1 + g_2 + g_3)$$

$$i_1 = -e\, J\,(b_1 + b_2 + b_3)$$

$(\theta + \theta_4) = \theta_5$ equals the angle between the primary voltage and primary current and $(\theta_3 + \theta_4)$ = the angle between the secondary current and the primary or impressed voltage. First let us calculate the total impedance of the circuit.

$$g_1 = \frac{100}{100^2 + 0} = \frac{1}{100} = .01 \qquad b_1 = 0$$

$$g_2 = 0 \qquad b_2 = -\frac{1}{1000} = -.001$$

$$g_3 = 98.4 \qquad b_3 = 82.0$$

$$G = g_1 + g_2 + g_3 = 98.41$$

$$Y_6 = G + J\,B$$

$$B = b_1 + b_2 + b_3 = 82.001$$

$$Y_6 = \sqrt{G^2 + B^2} = 128$$

$$Z_6 = \frac{1}{Y_6} = \frac{1}{128} = .00781 \qquad Z_6 = \sqrt{r_6 + x_6}$$

$$r_6 = .006$$

$$x_6 = .005$$

$$Z_6 + X = r_6 - r \quad J\,(x_6 + x)$$

$$r_6 + r = .006 + .001 = .007$$

$$x_6 + x =$$

Now $e = E_o - I_o\, Z_o$ =

Where Z_o = the impedance of the primary winding and load,

$$e = E_o - (i + j\,i_1)\,(R + J\,X) \quad \text{where}$$

R = resistance of primary + resistance of load.

X = reactance of primary + reactance of load.

$$e = E_o - (i\,R + J\,iX + J\,i_1\,R - i_1X)$$

$$= E - iR - JiX - Ji_1\,R + i_1X)$$

$$= E - iR + i_1X - J\,(ix + i_1R)$$

Again $E_o = e + (iR - i_1x) + (ix + i_1R)$

$$= e + ir - i_1x + J\,(ix + i_1R)$$

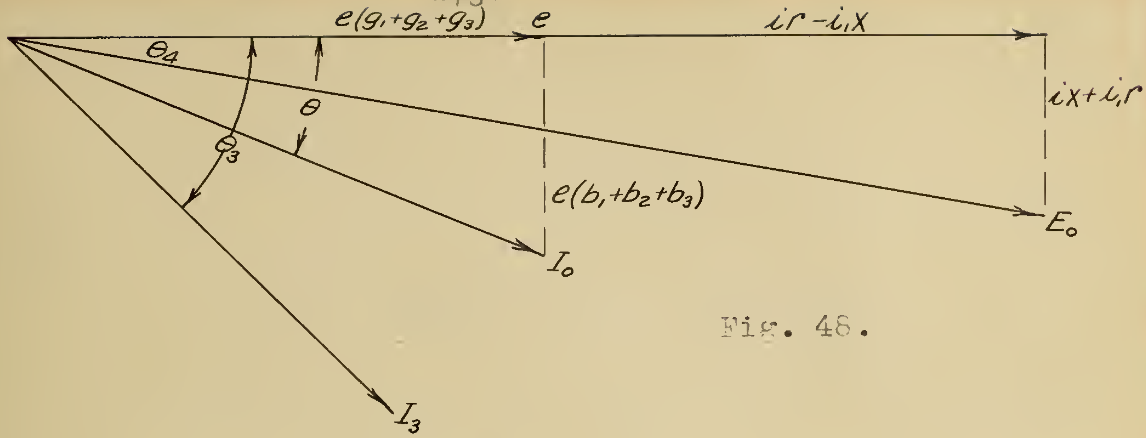


Fig. 48.

$$\text{Tan } \Theta_4 = \frac{ix + i_1R}{(iR - i_1x \text{ f } e)}$$

Table VIII has been derived by applying the above theory; the varying quantity being the load. A study of this table shows the extreme accuracy of the current transformer when worked over a very wide range of loads.

TABLE VIII

E_o	1.0	1.0	1.0	1.0	1.0	1.0	1.0
r_4	.1	.5	1.0	1.5	2.0	3.0	5.0
$r_o = r_6 + r_4 + r$	.107	.507	1.007	1.507	2.007	3.007	5.007
x_o	.01	.01	.01	.01	.01	.01	.01
$Z_o = r_o^2 + x_o^2$	.1074	.507	1.007	1.507	2.007	3.007	5.007
$I_o = \frac{E_o}{Z}$	9.31	1.972	.993	.663	.498	.332	.1995
$G + JB$	128	128	128	128	128	128	128
$e = I_o \div (G + JB)$	.0727	.0154	.00775	.00518	.00389	.00259	.00156
$I_3 = e\ X(g_3 + Jb_3)$	9.31	1.972	.993	.663	.498	.332	.1995
$\text{Tan}\ \theta_3$	-.8435	-.8435	-.8435	-.8435	-.8435	-.8435	-.8435
θ_3	$-40^\circ 9'$	$-40^\circ 9'$	$-40^\circ 9'$	$-40^\circ 9'$	$-40^\circ 9'$	$-40^\circ 9'$	$-40^\circ 9'$
$e\ (g1 + g2 + g_3) = i$	7.15	1.516	.763	.51	.383	.255	.1535
$e\ (b_1 + b_2 + b_3) = i_1$	5.96	1.263	.6355	.425	.319	.2123	.128
R	.101	.501	1.001	1.501	2.001	3.001	5.001
X	.005	.005	.005	.005	.005	.005	.005
i_1R	.722	.760	.764	.7655	.7662	.765	.768
i_1X	-.0298	-.00632	-.00317	-.0021	-.00159	-.1115	-.0006
$e + iR - i_1X$	.8245	.78172	.7749	.7728	.77168	.7687	.7702
ix	.0257	.00758	.00386	.00255	.001915	.001275	.000769
i_1R	-.602	-.633	-.636	-.638	-.638	-.647	-.6405
$(ix + i_1R)$	-.5963	-.6254	-.6231	-.6354	-.6360	-.6457	-.6397
$\text{Tan}\ \theta_4$	-.723	-.80	-.816	-.82	-.825	-.84	-.83
θ_4	$-35^\circ 52'$	$-38^\circ 40'$	$-39^\circ 13'$	$-39^\circ 21'$	$-39^\circ 32'$	$-40^\circ 2'$	$-39^\circ 42'$
$(\theta_3 - \theta)$	$-4^\circ 17'$	$-1^\circ 9'$	$-56'$	$-48'$	$37'$	$-17'$	$-27'$
$\theta = \theta_3$	$-40^\circ 9'$	$-40^\circ 9'$	$-40^\circ 9'$	$-40^\circ 9'$	$-40^\circ 9'$	$-40^\circ 9'$	$-40^\circ 9'$

VI TRANSFORMER WAVE SHAPES

It is a well established fact that the wave shape of the impressed e. m. f. has an appreciable effect upon the core losses of a transformer and in order to study this phenomena we will return to the simple method of obtaining the derivitive curve.

Consider the case of an ideal transformer in which the resistance may be considered as zero and assume the flux follows the sine law, then

$$\varphi = \Phi \sin \omega t$$

and the induced e.m.f. $e_1 = -N \frac{d\varphi}{dt}$

$$\frac{d\varphi}{dt} = + \Phi \omega \cos \omega t$$

if now,

$$-\frac{d\varphi}{dt} = - \Phi \omega \cos \omega t$$

and r is considered as zero, then the impressed e.m.f. e_1 will be equal to the induced e.m.f. e, or $e = e_1 = N \omega \Phi \cos \omega t$.

Now since in transformers of moderately large capacity the resistance is usually not greater than 1% it is evident that the above statements are true, for actual conditions, to an accuracy of one percent.

To find the derived curve of any given curve or wave and to check the accuracy of this method, assume an e.m.f. wave which follows the sine law and plot it to rectangular coordinates with such a scale that $E_{max.} = 1$. The equation of the original curve being $e = E_{max.} \sin \omega t$. Now since the flux curve is the

first derivitive of the e.m.f. curve and since the first derivitive is the rate of change of the original it is evident that if we can find the rate of change of e with respect to the time at any instant and plot this value at the instant for which it is calculated the result will be the flux curve produced by a sine wave of e.m.f. or vice versa if the original curve is taken as $\varphi = \Phi \sin \omega t$ the derived curve will be the impressed e.m.f. Let us assume equal intervals of time say ten degrees and find the change in flux during this interval of time, for our first step let us take -5° and $+5^{\circ}$, the change in flux will then be .174 and this value should be plotted on the zero of abscissae since it is the average rate of change during the interval of time from -5° to $+5^{\circ}$. It is also evident from the above discussion that when $\varphi = 0$ the e.m.f. induced is a maximum which is equivalent to saying that the flux curve has its maximum rate of change as the values pass through the zero point.

Hence we can multiply .174 by some constant K which will give unity for the maximum value of Φ or in the above case,

$$K = 5.74 \quad \text{and } K \times \varphi = 5.74 \times .174 = 1.0.$$

By continuing this process as illustrated above the following table number IX has been derived from which it is evident that the complete wave may be determined and it is also evident that for any given wave the derived curve may be determined. The percent error column in the table shows the accuracy of this method to be well within standard engineering practice. Curve sheet #9 page 81 shows these curves plotted scale.

TABLE IX

Time in 0^{o}	φ	φ	θ	K φ	Cos θ	Diff.	% Error
-5	-.087						
		.174	0	1	1	0	0
+5	+.087						
		.173	10	.993	.9848	+.008	.81
+15	.26						
		.163	20	.936	.939	-.003	.32
+25	.423						
		.147	30	.844	.866	-.022	.25
+35	.570						
		.137	40	.786	.766	+.02	.26
+45	.707						
		.111	50	.637	.642	-.005	.78
+55	.818						
		.088	60	.505	.500	+.005	1.00
+65	.906						
		.06	70	.344	.342	+.002	.59
+75	.966						
		.03	80	.172	.173	-.001	.58
+85	.996						
		0	90	0	0	0	0
+105	.996						

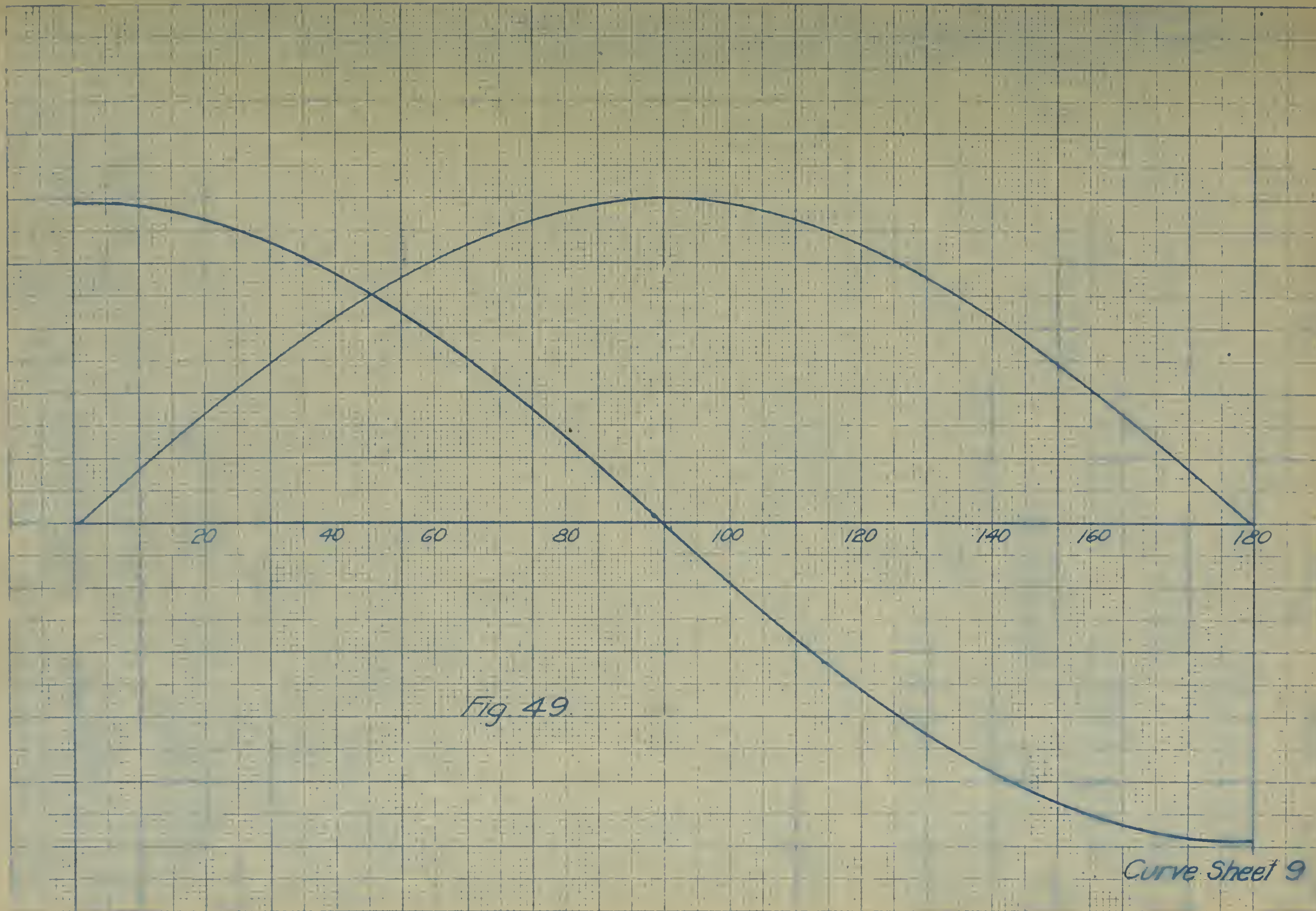


Fig. 49.

Curve Sheet 9

Curves of irregular shapes may be determined by the above precess. Table X shows the results obtained when it is desired to find the instantaneous values of the induced e.m.f. The flux wave being represented by the equation,

$$\varphi = \Phi \sin \theta + .33\, \Phi \sin (3\,\theta - 270^{\circ})$$

Curve sheet #10 page 84 shows the two waves plotted to scale.

It is a well known fact that the hysteresis loss is proportional very approximately to the 1.6 power of the flux density. It is, therefore, desirable to find the relation existing between the e.m.f. wave and the flux especially the maximum flux.

Neglecting the small I R drop which is out of phase with respect to the impressed voltage, we have

$$e = K \frac{d\varphi}{dt} = K_1 \frac{d\beta}{dt} \qquad \text{(a)}$$

where e is the instantaneous value of the impressed e.m.f. and β is the corresponding flux density.

Therefore

$$e\ dt = K_1\ d\beta$$

and

$$\int_{t_1}^{t_2} e\ dt = K \int_{\beta = 0}^{\beta = \text{max.}} d\beta \qquad \text{(b)}$$

or

$$e \text{ average } (t_2 - t_1) = K \beta \text{ max.} \qquad \text{(c)}$$

where t_1 is the time at the beginning of the integration and t_2 is the time at the end, $(t_2 - t_1)$ representing the time taken for the flux wave to pass from zero to maximum value. Equation (c) expresses the fact that the maximum density is proportional to the area of the e.m.f. curve between t_1 and t_2 . Now the flux curve

TABLE X

θ	Sin θ	Sin (3θ - 270)	.33 Sin(3θ - 270)	φ	θ	e	Ke
-5	-.087	.965	.318	.231	0	.174	.542
5	.087	.965	.318	.405	10	.086	.268
15	.258	.707	.233	.491	20	.016	.050
25	.422	.258	.085	.507	30	-.019	-.0592
35	.573	-.258	-.085	.488	40	-.014	-.0436
45	.707	-.707	-.233	.474	50	+.024	+.0748
55	.816	-.965	-.318	.496	60	.090	.2806
.65	.906	-.965	-.318	.588	70	.144	.449
.75	.965	-.707	-.233	.732	80	.179	.558
.85	.996	-.258	-.085	.911	90	.170	.530
.95	.996	.258	.085	1.081	100	.117	.365
105	.965	.707	.233	1.198	110	.026	.081
115	.906	.965	.318	1.224	120	-.086	-.268
125	.816	.965	.318	1.138	130	-.198	-.617
135	.707	.707	.233	.940	140	-.282	-.879
145	.573	.258	.085	.658	150	-.321	-1.00
155	.422	-.258	-.085	.337	160	-.312	-.972
165	.258	-.707	-.233	.025	170	-.256	-.748
175	.087	-.965	-.318	-.231	180	-.174	-.543
185	-.087	-.965	-.318	-.405			

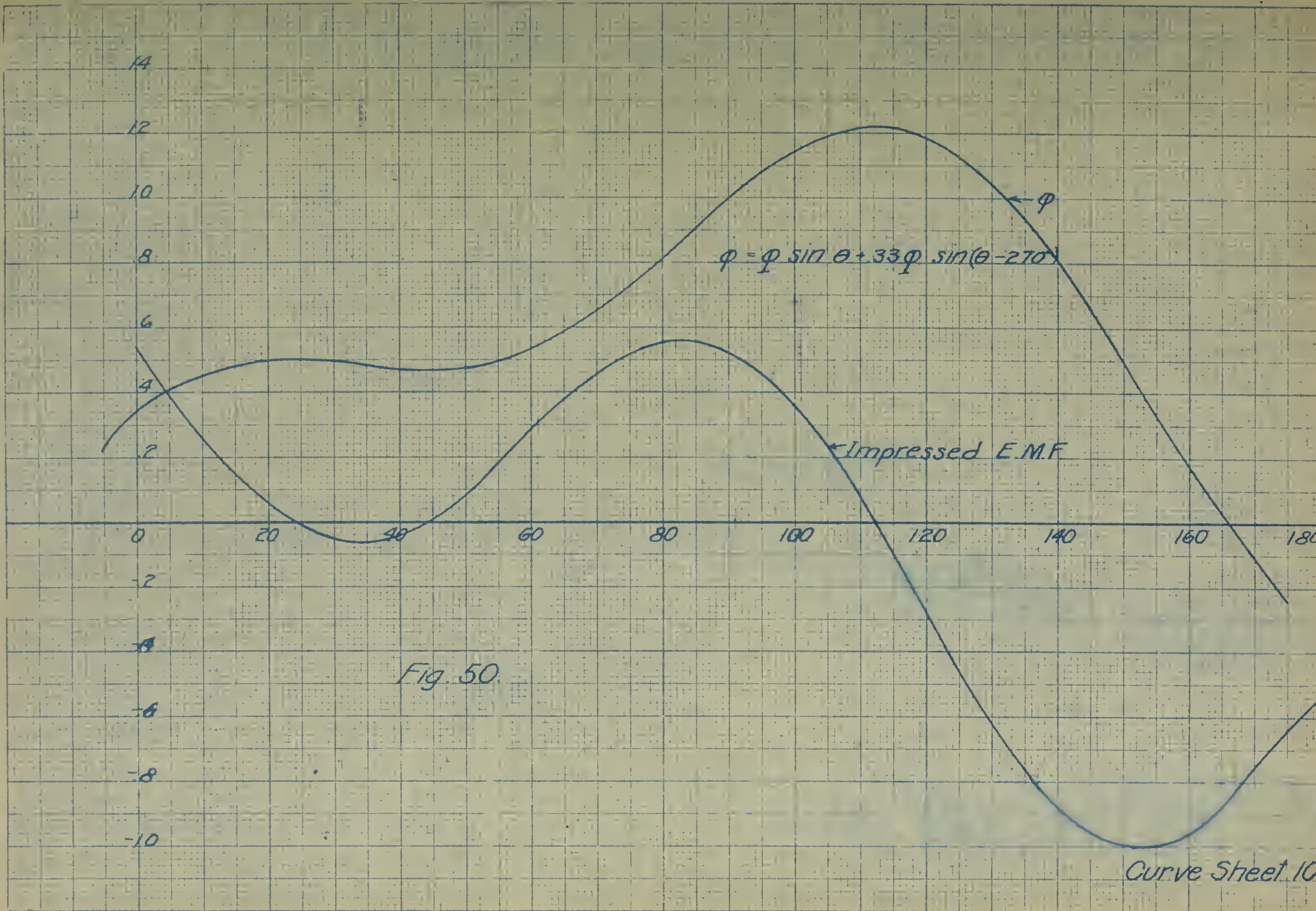


Fig. 50.

Curve Sheet 10

curve is at its maximum point when the e.m.f. curve passes through zero, and in an alternating current circuit as much flux must enter the core as leaves. It follows, therefore, from figure 51 that the positive area expressed by e $(t_2 - t_1)$ of the e.m.f. wave must equal the negative area of the e.m.f. wave between t_2 and t_3, where t_3 is the point of maximum negative e.m.f.

Hence the area to the left of the maximum ordinate must equal that to the right of the maximum, and the area of the e.m.f. wave from t_1 to t_2 is proportional to the maximum flux density.

Since it is often desirable to draw the wave of flux when the e.m.f. wave is known the following method will be given.

Bisect the area of the e.m.f. wave and plot β ordinates proportional to the areas A_1, $a_1 + a_2$, + ----, as shown in figure . The sum of all these areas will be e $(t_2 - t_1)$ and is proportional to the maximum flux density. Which shows again that one half the area of the e.m.f. wave is proportional to the maximum flux density. The following Table XI and curve sheet 12 gives an example of a given e.m.f. wave from which the flux wave was determined.

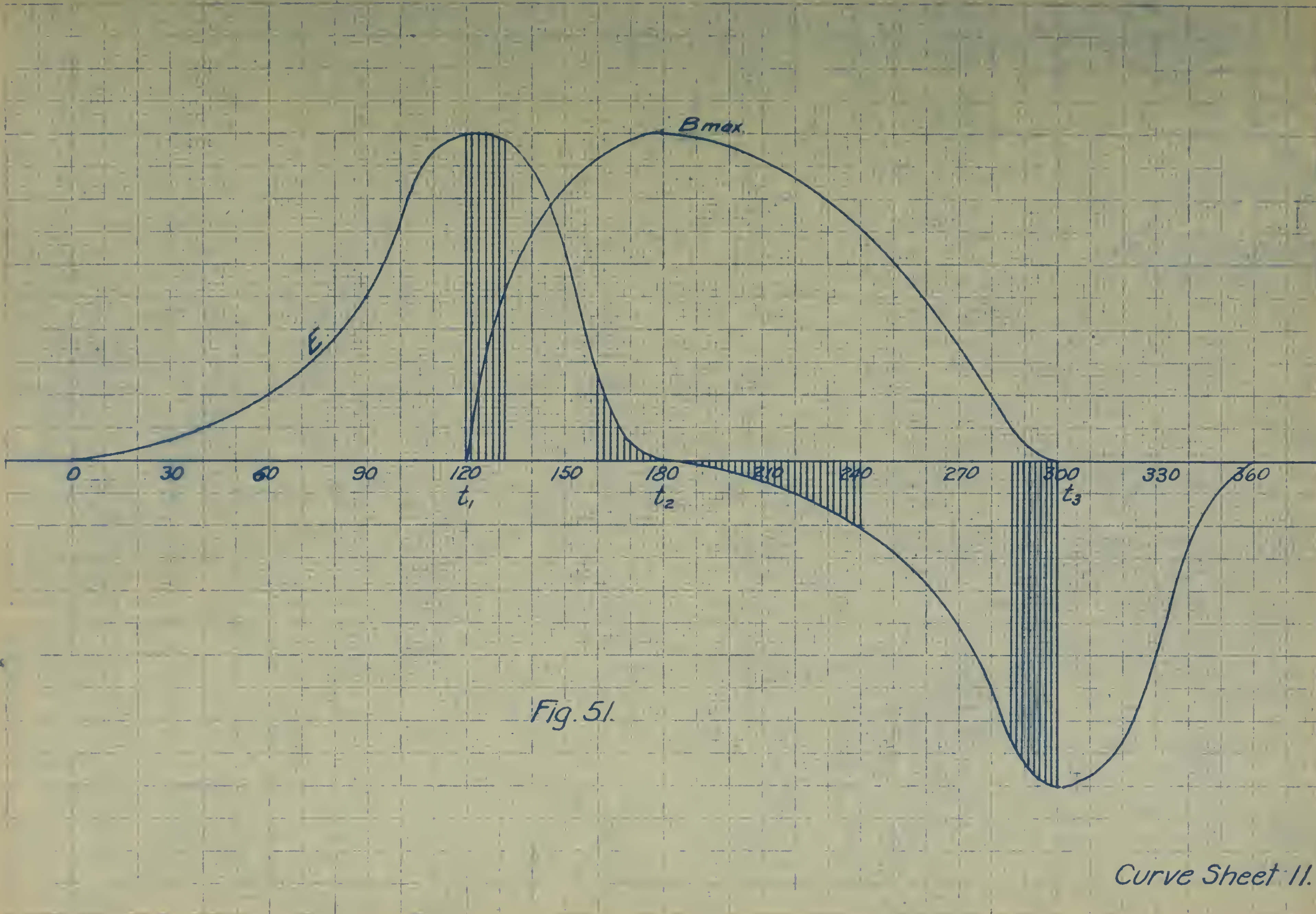


Fig. 51.

Curve Sheet II.

TABLE XI

e.m.f. Curve		To find T_1			To determine ϕ curve	To find ϕ	$K = .0267$	
θ^{o}	E.M.F.	θ	ΔA	A	θ	area	area	area
0	0	0	0	0	125^{o}	7.76	7.76	.272
10	.038	5	.05	.05	135	9.23	16.99	.454
20	.051	15	.38	.43	145	7.95	24.94	.655
30	.077	25	.51	.94	155	6.15	31.09	.829
40	.10	35	.77	1.71	165	4.1	35.19	.939
50	.15	45	1.0	2.71	175	2.0	37.19	.991
60	.256	55	1.5	4.21	180	.25	37.44	1.
70	.308	65	2.56	6.77	185	-.05	37.39	.997
80	.435	75	3.08	9.85	195	-.38	37.01	.988
90	.564	85	4.35	14.20	205	-.51	36.5	.975
100	.692	95	5.64	19.84	215	-.77	35.73	.954
110	.845	105	6.92	26.76	225	-1.0	34.73	.941
120	1.0	115	8.45	35.21	235	-1.5	33.23	.888
130	.923	125	.10	45.21	245	-2.56	30.67	.82
140	.795	135	9.23	54.44	255	-3.08	27.59	.736
150	.615	145	7.95	62.39	265	-4.35	23.24	.621
160	.41	155	6.15	68.54	275	-5.64	17.60	.47
170	.20	165	4.1	72.64	285	-6.92	10.68	.285
180	0	175	2.0	74.64	295	-8.45	2.23	.0595
		180	.25	74.89	297.24	-2.24	0	0

$$\text{Half area} = \frac{74.89}{2} = 37.45$$

At $\theta = 115$ area $= 35.21$ ΔA = area

$\theta = 125$ area $= 45.21$ A = total area.

Diff. 10^{o} Diff. 10

At $\theta = 115$ area $= 35.21$

$\theta = ?$ area $= 37.45$

Diff.= ? Diff. 2.24

$$\frac{x}{10} = \frac{2.24}{10} = 2.24$$

$$\theta = 115 + 2.24 = 117.24^{o} = T_1$$

Therefore area at $125^{o} = (45.21 - 37.45) = 7.76$

$$117.24^{o} + 180 = 297.24^{o} = T_2 .$$

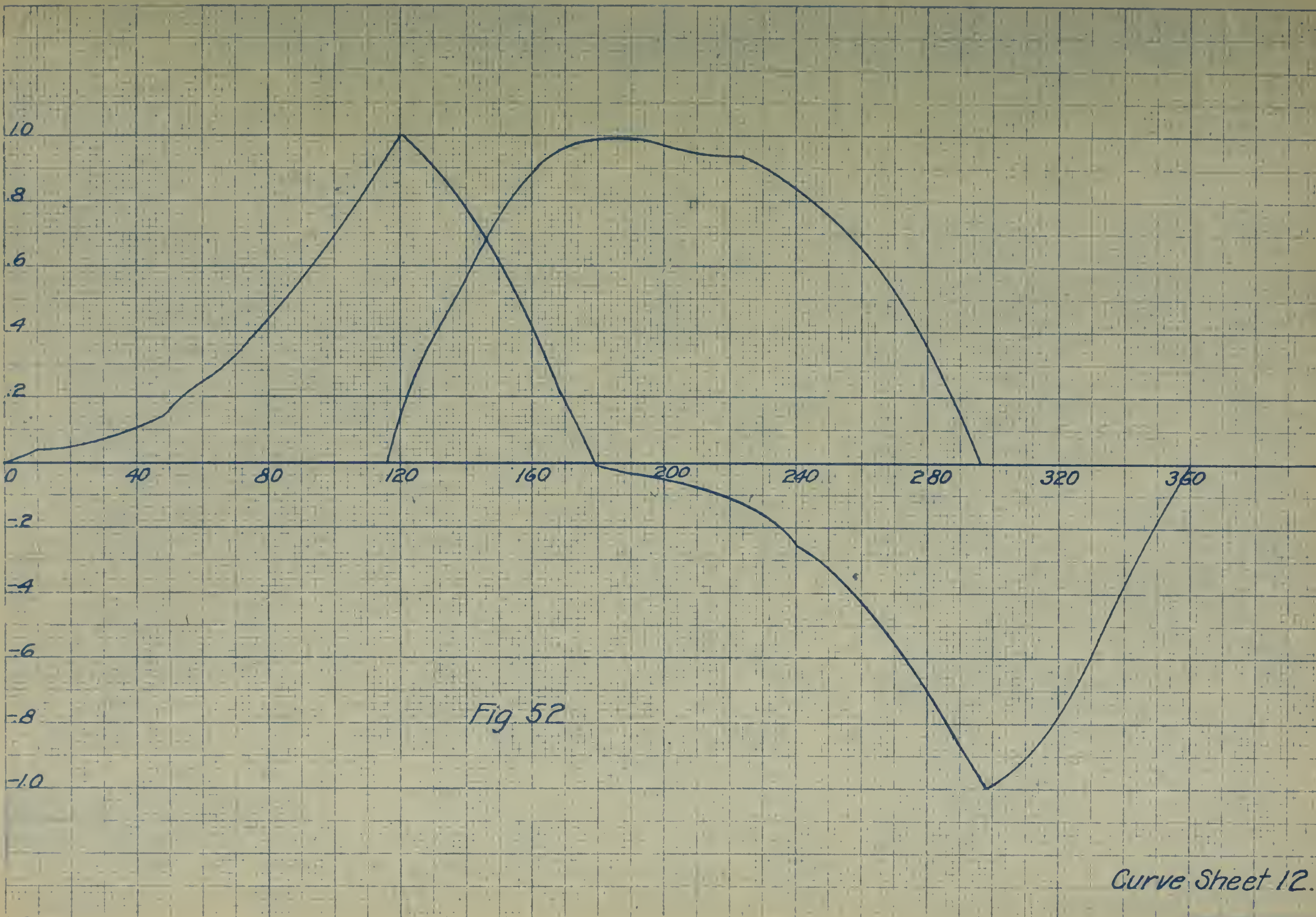


Fig 52

Curve Sheet 12.

EXCITING CURRENT OF TRANSFORMERS

When data is available for determining the hysteresis loop, it is possible to find the shape of the exciting current wave for any wave of e.m.f. or for any given form of exciting current the flux wave and impressed wave of e.m.f. may be determined. Given the following data on the hysteresis loop of a commercial transformer.

TABLE XII

For ascending values of flux.

φ	I
0	.50
.34	.60
.58	.70
.77	.80
1.00	1.00

For descending values of flux.

φ	I
1.00	1.00
.99	.80
.96	.60
.91	.40
.85	.20
.75	0

.56	-.20
.25	-.40
0	-.50

Determine the current wave with a sine wave of impressed e.m.f.

It is evident from previous discussions that the flux wave, which is produced by a sine wave of impressed e.m.f. is also a sine wave and lags behind the impressed e.m.f. by 90 electrical degrees, while the induced lags behind the flux wave by 90 electrical degrees. Figure 53 shows these waves, also the hysteresis loop and exciting current produced by a sine wave of e.m.f.

To determine the exciting current having given the hysteresis loop and e.m.f. wave. First find the flux wave using the method previously given which applies, since

$$e = N \frac{d\varphi}{dt}$$

$$K \int e \, dt = \int d\varphi = \underline{\varphi}$$

Having determined the flux wave the exciting current may be found as follows. Choose any value of flux as, ab, Fig. 53. The corresponding value of flux on the hysteresis loop is given by, cd, and the current corresponding to β = cd, is found to be, oa. Now plot oa, to some predetermined scale, on the line ab which in figure 53 is represented by ae and is the proper value of $I_{ex.}$ for the flux density which was taken. In this manner all points on the current curve may be located.

Again with a sine wave of exciting current flowing the shape of the flux wave produced in the core can be determined and

from this wave the impressed or induced e.m.f. wave is derived as shown in figure 54. The data was obtained by the reverse process of the method given above.

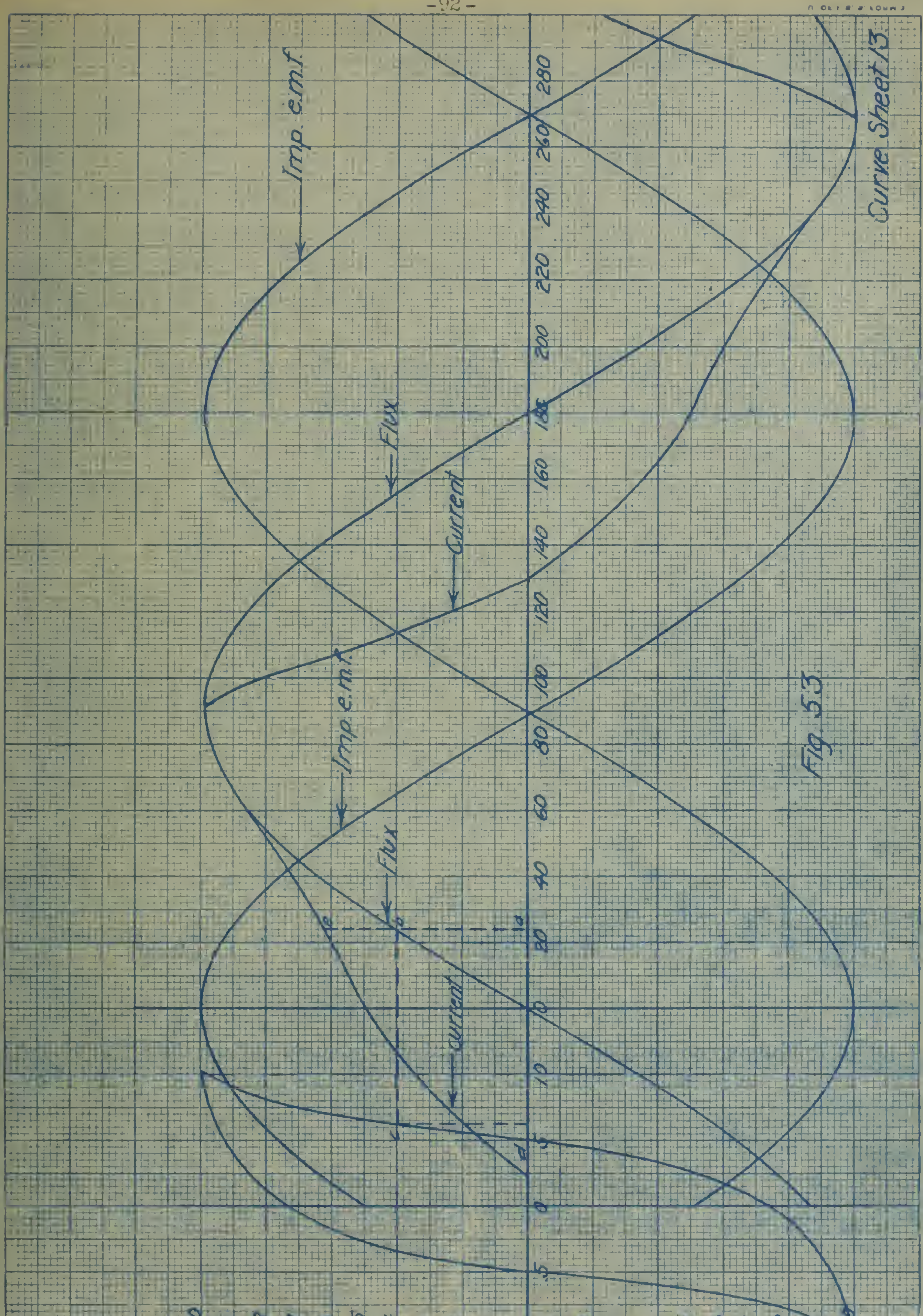

Imp. e.m.f.
Imp. e.m.f.
Flux
Flux
Current
Current
Curve Sheet 13
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
10
.9
.8
.7
.6
.5
.4
.3
.2
.1

Fig. 53

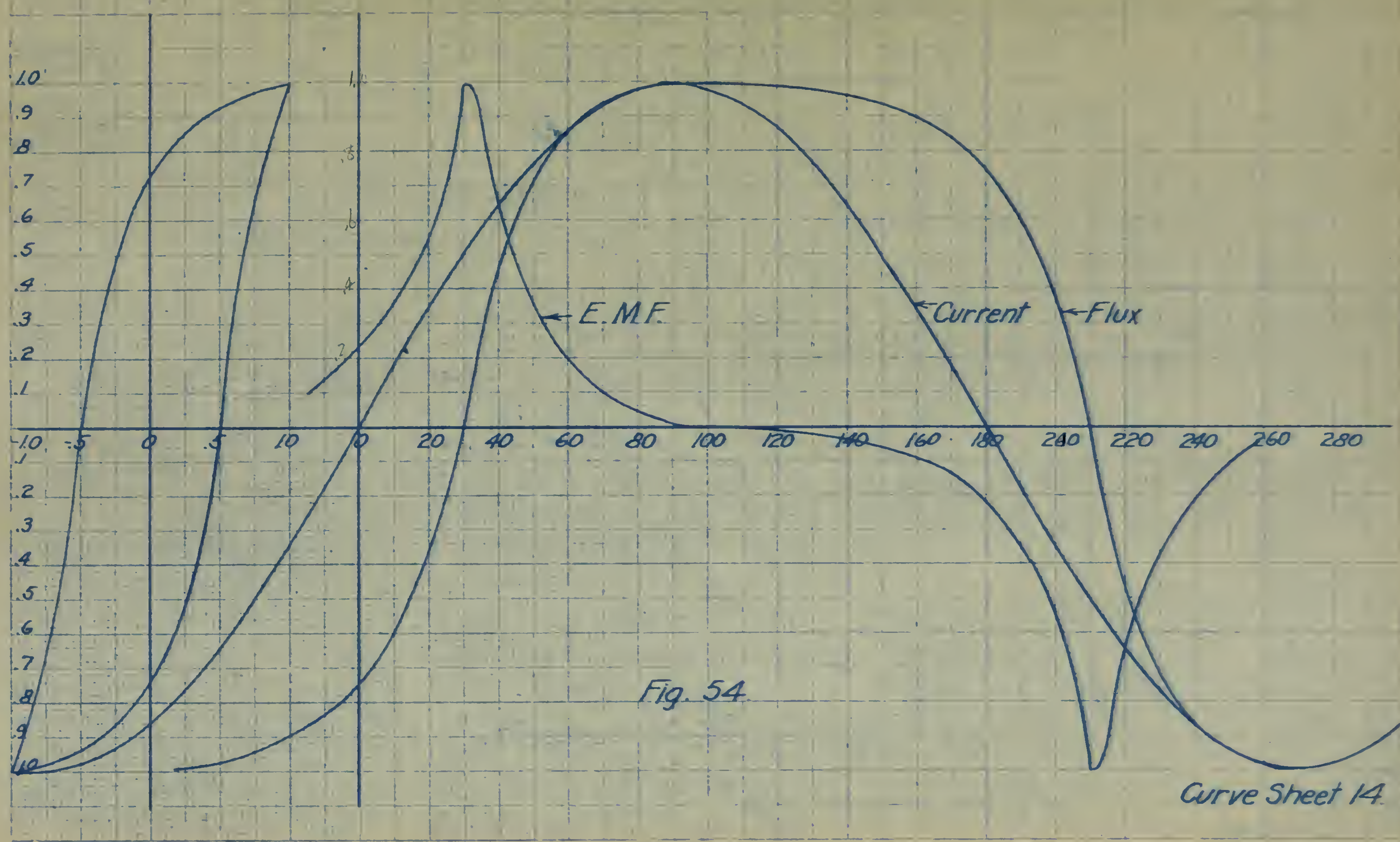


Fig. 54

Curve Sheet 14

TABLE XIII

θ	φ	φ	e = K φ	θ
28	-.09	.21	1.00	30
32	+.12	.20	.952	34
36	.32	.15	.714	38
40	.47	.12	.571	42
44	.59	.10	.476	46
48	.69	.08	.381	50
52	.77	.06	.285	54
56	.83	.04	.1905	58
60	.87	.04	.1905	62
64	.91	.03	.1428	66
68	.94	.02	.0952	70
72	.96	.015	.0714	74
76	.975	.01	.0476	78
80	.985	.01	.0476	82
84	.995	.005	.0238	86
88	.999	.001	.0047	90
92	1.00	0	0	94
96	1.00	0	0	98
100	1.00	0	0	102
104	1.00	.001	.0476	106
108	.999	.001	.0476	110
112	.998	.002	.00952	114
116	.996	.003	.0143	118
120	.993			

θ	φ	φ	e = K φ	θ
120	.993	.003	.0143	122
124	.980	.003	.0143	126
128	.987	.005	.0238	130
132	.982	.006	.0286	134
136	.976	.008	.0381	138
140	.968	.012	.0571	142
144	.956	.013	.062	146
148	.943	.013	.062	150
152	.930	.016	.076	154
156	.914	.018	.0858	158
160	.896	.018	.0858	162
164	.880	.021	.100	166
168	.859	.029	.138	170
172	.830	.035	.167	174
176	.795	.045	.212	178
180	.750	.05	.238	182
184	.700	.065	.31	186
188	.635	.075	.357	190
192	.56	.090	.428	194
196	.47	.100	.476	198
200	.37	.130	.62	202
204	.24	.150	.715	206
208	.09	.21	1.00	210
216	-.12	.18		

K = 4.762

WAVE ANALYSIS

It is often of very great importance to know to what extent higher harmonics enter into the e.m.f. or current waves of transformers or generators and by the application of "Fouriers Series" this phenomena may be investigated.

A great many curves having irregular shapes, may be expressed in terms of sines and cosines and the general expression for a function may be written,

$$Y = A_0 + A_1 \sin \varphi + A_2 \sin 2\varphi + A_3 \sin 3\varphi + \text{-----}$$
$$\text{--}A_n \sin n\varphi + B_1 \cos \varphi + B_2 \cos 2\varphi + B_3 \cos 3\varphi + \text{---}B_n \cos n\varphi.$$

The conditions which the function (y) must satisfy are:-

(a) That is shall be single valued.

(b) That it shall have only a finite number of maximum and minimum points.

(c) That it shall be continuous.

Since in general the current or e.m.f. wave from a generator or transformer satisfy these conditions it is evident that such waves may be expressed by the general equation as given above. Therefore, a method must be found by which A_0, A_1, A_2, A_3--------- B_1, B_2, B_3 -----B_n may be determined.

First investigate the value of A_0; to do this multiply the general equation as given above by $d\varphi$ and integrate between the limits zero and 2π.

Thus,

$$\int_0^{2\pi} Y\, d\varphi = \int_0^{2\pi} A_0\, d\varphi + \int_0^{2\pi} A_1 \sin \varphi\, d\varphi + \int_0^{2\pi} A_2 \sin 2\varphi\, d\varphi + \int_0^{2\pi} A_3 \sin 3\varphi\, d\varphi$$

$$+ \text{---} \int_0^{2\pi} A\, n \sin n\varphi \, d\varphi + \int_0^{2\pi} B_1 \cos \varphi \, d\varphi + \int_0^{2\pi} B_2 \cos 2\varphi \, d\varphi + \int_0^{2\pi} B_3 \cos$$

$$3\varphi \, d\varphi + \text{-----} \int_0^{2\pi} B_n \cos n\varphi \, d\varphi.$$

Since $A\, n \sin n\varphi \, d\varphi$ is the general sine term the integral of this expression should equal the integral of all sine terms or,

$$\int_0^{2\pi} A\, n \sin n\varphi \, d\varphi = \frac{An}{n} \Big[-\cos n\varphi \Big]_0^{2\pi} = 0$$

hence all sine terms will be zero when integrated between the limits zero and 2π.

Applying the same reasoning to the cosine terms of the above expression it is evident that,

$$\int_0^{2\pi} Bn \cos n\varphi \, d\varphi = 0$$

and hence all cosine terms will be zero and there will be left in the general expression,

$$\int_0^{2\pi} y \, d\varphi = \int_0^{2\pi} A_0 \, d\varphi = A_0 \varphi \Big]_0^{2\pi} = 2\pi A_0.$$

or $A_0 = \frac{1}{2\pi} \int_0^{2\pi} y \, d\varphi.$

But since $\int_0^{2\pi} y \, d\varphi$ is the area of the original curve it is evident that A_0 equals the average ordinate of the original curve or

$$A_0 = \frac{1}{2\pi} \int_0^{2\pi} y \, d\varphi.$$

Now since the positive lobe of an electrical wave is equal to the

negative lobe it is evident their sum is zero and therefore A_0 is zero as may be seen from the figure given below.

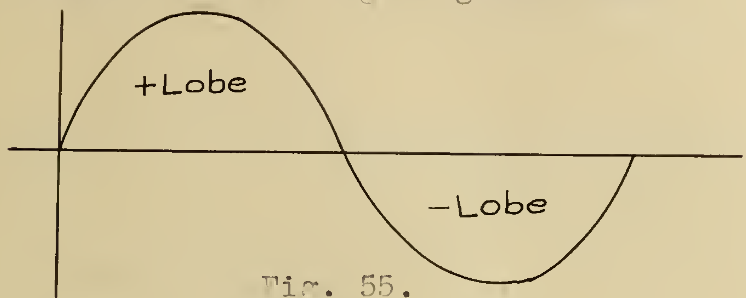


Fig. 55.

When the neutral axis is displaced from the reference axis A_0 will have a positive or negative value as indicated below.

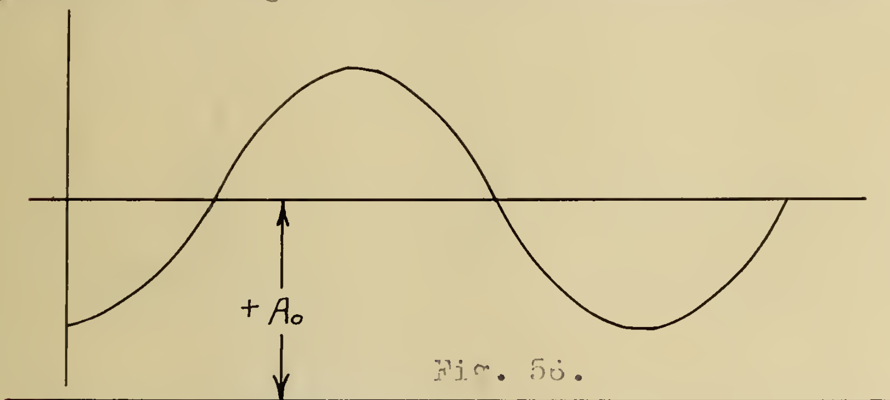


Fig. 56.

From the above discussion it is seen that A_0 is in general equal to zero for any wave as produced by an electric generator or transformer and will not be mentioned further.

To find A_1 multiply the general equation by $\sin\varphi\, d\varphi$ and integrate between the limits zero and 2π, or

$$\int_0^{2\pi} y \sin\varphi\, d\varphi = \int_0^{2\pi} A_1 \sin\varphi \sin\varphi\, d\varphi + \int_0^{2\pi} A_2 \sin 2\varphi \sin\varphi\, d\varphi + \int_0^{2\pi} A_3$$

$$\sin 3\varphi \sin\varphi\, d\varphi + \int_0^{2\pi} A_n \sin n\varphi \sin\varphi\, d\varphi + \text{-----} \int_0^{2\pi} B_1 \cos\varphi \sin\varphi\, d\varphi$$

$$+\int_0^{2\pi} B_2 \cos 2\varphi \sin \varphi \, d\varphi + \int_0^{2\pi} B_3 \cos 3\varphi \sin \varphi \, d\varphi + \int_0^{2\pi} B_n \cos n\varphi \sin \varphi \, d\varphi.$$

It is now necessary to investigate the general sine term and if its value is zero then all sine terms will be zero except the first which in reality is,

$$\int_0^{2\pi} A_1 \sin^2 \varphi \, d\varphi$$

hence from the general sine term we have

$$\int_0^{2\pi} A_n \sin n\varphi \sin \varphi \, d\varphi.$$

A general solution for $\int_0^{2\pi} \sin n\varphi \sin \varphi \, d\varphi$ will be given here and for any other similar integration the same method can be applied.

$$\cos (n\varphi + \varphi) = \cos n\varphi \cos \varphi - \sin n\varphi \sin \varphi$$

$$\cos (n\varphi - \varphi) = \cos n\varphi \cos \varphi + \sin n\varphi \sin \varphi$$

$$-\cos (n\varphi + \varphi) + \cos (n\varphi - \varphi) = 2 \sin n\varphi \sin \varphi$$

$$\sin n\varphi \sin \varphi = \frac{1}{2} \left[\cos (n\varphi - \varphi) - \cos (n\varphi + \varphi)\right]$$
$$= \frac{1}{2} \left[\cos (n - 1) \varphi - \cos (n + 1) \varphi\right]$$

Therefore

$$\int_0^{2\pi} \sin n\varphi \sin \varphi \, d\varphi = \frac{1}{2}\int_0^{2\pi} \cos (n-1)\varphi - \frac{1}{2}\int_0^{2\pi} \cos (n+1)\varphi = 0$$

from the above it is evident that all the sine terms except the first will be zero. The general sine cosine term is $B_n \cos n\varphi \sin \varphi \, d\varphi$ and if this term is zero when integrated as indicated above it is evident that all sine cosine terms will be zero.

$$\int_0^{2\pi} Bn \cos n\varphi \sin \varphi \, d\varphi$$

It can easily be shown that,

$$\cos n\varphi \sin \varphi = 1/2 \left[\sin (n+1) \varphi - \sin (n-1) \varphi \right]$$

and

$$Bn \int_0^{2\pi} \cos n\varphi \sin \varphi \, d\varphi = \frac{Bn}{2} \int_0^{2\pi} \sin (n+1) \varphi - \frac{Bn}{2} \int_0^{2\pi} \sin (n-1) \varphi \, d\varphi$$

$$= \text{zero.}$$

Therefore all B terms will go to zero leaving in the general expression,

$$\int_0^{2\pi} y \sin \varphi \, d\varphi = \int_0^{2\pi} A_1 \sin^2 \varphi \, d\varphi$$

Since $\sin^2 \varphi = \frac{1}{2} - \frac{1}{2} \cos 2\varphi$

$$\int_0^{2\pi} A_1 \sin^2 \varphi \, d\varphi = \int_0^{2\pi} \frac{A_1}{2} \, d\varphi - \int_0^{2\pi} \frac{A_1}{2} \cos 2\varphi \, d\varphi = \pi A_1 \quad \text{and}$$

$$\int_0^{2\pi} Y \sin \varphi \, d\varphi = \pi A_1 \quad \text{or}$$

$$\frac{1}{2\pi} \int_0^{2\pi} Y \sin \varphi \, d\varphi = \frac{A_1}{2}$$

which shows that A_1 is 2 times the mean ordinate of the curve represented by $Y \sin \varphi$. To find the value of B_1 multiply the original equation by $\cos \varphi \, d\varphi$ which gives,

$y \cos \varphi \, d\varphi = A_1 \sin \varphi \cos \varphi \, d\varphi + A_2 \sin 2\varphi \cos \varphi \, d\varphi + A_3 \sin 3\varphi \cos \varphi \, d\varphi +$ ------- $A_n \sin \varphi \cos \varphi \, d\varphi + B_1 \cos^2 \varphi \, d\varphi + B_2 \cos 2\varphi \cos \varphi \, d\varphi + B_3 \cos 3\varphi \cos \varphi \, d\varphi +$ ------- $B_n \cos n\varphi \, d\varphi$. Now if this expression is integrated between the limits zero and 2π it is

found that all terms will be zero except,

$$\int_0^{2\pi} Y \cos \varphi \, d\varphi = \int_0^{2\pi} B_1 \cos^2 \varphi \, d\varphi$$

from which expression it is seen that,

$$\pi B_1 = \int_0^{2\pi} y \cos \varphi \, d\varphi$$

or,

$$\frac{B_1}{2} = \int_0^{2\pi} \frac{y \cos \varphi \, d\varphi}{2\pi}$$

or B_1 = 2 times the mean ordinate of the curve represented by y cos φ.

Now, since a conductor on an alternator receives the same induction when under a north pole as when passing under a south pole, it is evident that the even harmonics must not be present. If this be true the value of A_2 , A_4 , A_6 ----------- B_2, B_4, B_6------- etc. must be zero and only the odd harmonics need be considered.

To find A_3 multiply the general expression by sin 3φ dφ and integrate between the limits zero and 2π. Again it will be found that all terms will be zero except the following,

$$\int_0^{2\pi} y \sin 3\varphi \, d\varphi = \int_0^{2\pi} A_3 \sin^2 3\varphi \, d\varphi$$

$= \pi A_3$ or A_3 is 2 times the mean ordinate of the curve represented by the equation y sin 3φ.

In like manner it can be shown that B_3 = 2 x the mean ordinate of the curve represented by y cos 3φ dφ.

The same process is used to find A_5 and B_5, A_7 and B_7, A_9 and B_9, or so many coefficients as desired.

Now since $A_1 \sin \varphi$ and $B_1 \cos \varphi$ are waves of the same frequency, they may be combined in one equation,

$$Y_1 = A_1 \sin \varphi + B_1 \cos \varphi$$

and let $A_1 = C_1 \sin \omega$, $B_1 = C_1 \cos \omega$. Now since A_1 is a constant for any particular case it is evident that $C_1 \sin \omega$ and $\cos \omega$ must be constants.

$$Y_1 = C_1 \sin \omega \sin \varphi + C_1 \cos \omega \cos \varphi$$

$$Y_1 = C_1 \cos (\omega - \varphi)$$

Therefore if C_1 and ω are known the combined fundamental waves which make up the total fundamental, or first harmonic, can be plotted.

From above, $A_1^2 = C_1^2 \sin^2 \omega$ and $B_1^2 = C_1^2 \cos^2 \omega$.

$$C_1 = \sqrt{A_1^2 + B_1^2}$$

Again $\frac{C_1 \sin \omega}{C_1 \cos \omega} = \frac{A_1}{B_1} = \tan \omega$ and from the above expressions C_1 and ω can be determined. ω is of course the phase displacement of the fundamental from the original curve.

If A_1 and B_1 are both positive in sign then ω must be in the first quadrant or must have a value between zero and 90°. If A_1 and B_1 are both negative in sign then ω must be in the third quadrant. If A_1 is negative and B_1 is positive, ω must be in the fourth quadrant and finally if A_1 is positive in sign and B_1 is negative then ω must be in the second quadrant.

$$Y_3 = A_3 \sin 3\varphi + B_3 \cos 3\varphi.$$

$$C_3 = \sqrt{A_3^2 + B_3^2} \qquad \tan \omega_3 = \frac{A_3}{B_3}$$

and,

$$Y_5 = A_5 \sin 5\varphi + B_5 \cos 5\varphi$$

$$C_5 = \sqrt{A_5^2 + B_5^2} \qquad \tan \omega_5 = \frac{A_5}{B_5}$$

$$Y_n = A_n \sin n\varphi + B_n \cos n\varphi$$

$$C_n = \sqrt{A_n^2 + B_n^2} \qquad \tan \omega_n = \frac{A_n}{B_n}$$

Since the addition to two sine curves of the same frequency will give a resultant sine curve of the same frequency it is evident that the curve represented by,

$Y_1 = A_1 \sin \varphi + B_1 \cos \varphi$ is correctly represented by,

$$Y_1 = C_1 \cos (\omega - \varphi) \quad \text{and}$$

$$Y_3 = C_3 \cos (\omega - 3\varphi) \quad \text{etc.}$$

These curves which represent the various harmonics can then be plotted as one curve since all the terms may be determined when A_1, A_3, A_5----- A_n; B_1, B_3, B_5----- B_n have been determined and the original curve will be the sum of the various components, or

$$Y = Y_1 + Y_3 + Y_5 + Y_7 + \text{--------} \; Y_n$$

To analyze a wave, we first find the instantaneous values of same by means of the oscillograph or otherwise, through at least a range of 180 electrical degrees. But since the wave has two lobes one + and the other - and both symmetrical with respect to the axis we need consider only one lobe and by going through the previous theory it may be seen that the values of A_{n1}, B_n etc. hold for 180° as well as for 360°.

Now with the instantaneous values of the curve known, take values every ten degrees or as often as the accuracy of the

problem demands and make columns of the following,

(a) Angular displace of time.

(b) Column of values of ordinates of data curve = F.

(c) Sin φ.

(d) F sin φ.

(e) cos φ.

(f) F cos φ.

(g) Sin 3φ.

(h) Cos 3φ.

(i) F sin 3φ.

(j) F cos 3φ.

etc. for as many harmonics as desired.

As is seen the even harmonics (F cos 2φ and F sin 2φ and F cos 4φ and F sin 4φ etc.) are omitted since in an electrical machine such as a generator or transformer these harmonics very seldom exist.

Now if the ordinates are taken every 10° over the 180° there will be 19 ordinates and since the values at 0° and 180° are equal, take the average readings of 0° and 180° add these to the other 17 and divide by 18. Hence the average of F cos φ, for instance, would be (1/2(zero reading + 180° reading) + the other 17 readings) ÷ 18.

Since A_n = Z average of F sin nφ

Average F sin nφ = 1/2 A_n

Also B_n = Z average of F cos nφ

Average F cos nφ = 1/2 B_n

which is the general expression for any harmonic, hence by giving

$$\tan \omega_1 = \frac{.396}{.2016} = 1.930 = \frac{B_1}{A_1}$$

n any positive integral value we may investigate as many harmonics as desired.

The following data for an e.m.f. and current curve has been obtained from oscillograph records and from this data the various harmonics have been found as illustrated on the following pages.

φ	e	i
0	.245	.50
10	.390	.55
20	.650	.60
30	1.00	.67
40	.740	.73
50	.450	.80
60	.240	.87
70	.128	.94
80	.054	.99
90	00	1.0
100	-.0064	.80
110	-.011	.50
120	-.015	.23
130	-.025	.03
140	-.043	-.14
150	-.072	-.26
160	-.098	-.36
170	-.165	-.44
180	-.246	-.5

For the e.m.f. curve,

$$Y_1 = \sqrt{\overline{2016}^2 + \overline{39}^2} \qquad \tan \omega_1 = \frac{.2016}{.390} = \omega_1 = 27^\circ\ 21'$$

$$Y_3 = \sqrt{\overline{248}^2 + \overline{.012}^2} \qquad \tan \omega_3 = -\frac{-.248}{-.012} = \omega_3 = +87^\circ\ 14'$$

$$Y_5 = \sqrt{\overline{.04}^2 + \overline{.116}^2} \qquad \tan \omega_5 = \frac{-.04}{-.116} = \omega_5 = +161^\circ$$

For the current curve,

$$Y_1 = \sqrt{\overline{.710}^2 + \overline{.538}^2} \qquad \tan \omega_1 = \frac{.710}{.538} = 52^\circ\ 52'$$

$$Y_3 = \sqrt{\overline{.186}^2 + \overline{.042}^2} \qquad \tan \omega_3 = \frac{-.186}{-.042} = 77^\circ\ 18'$$

$$Y_5 = \sqrt{\overline{.102}^2 + \overline{.002}^2} \qquad \tan \omega 5 = \frac{-.102}{-.002} = -88^\circ\ 53'$$

By adding the values of y_1, y_3 and y_5 in their proper phase relation it will be found that the e.m.f. wave has more than

these 3 harmonics and must be still further analyzed in order to determine the complete mathematical equation. While in the current curve the harmonics above the 5th are very small and of no particular value.

TABLE XIV

Angle	F	Sine θ	F sin θ	Cos θ	F cos θ	Sin 3θ	F sin 3θ	Cos 3θ	F cos 3θ	Sin 5θ	F sin 5θ	Cos 5θ	F cos 5θ
0	.245	0	0	1.0	.245	0	0	1.0	.245	0	0	1.0	.245
10	.39	.174	.0679	.984	.384	.5	.195	.866	.338	.766	.299	.643	.251
20	.65	.342	.222	.939	.61	.866	.563	.5	.325	.984	.639	-.174	∓133
30	1.0	.5	.5	.866	.866	1.0	1.0	0	0	.5	.5	-.866	∓866
40	.74	.643	.476	.766	.567	.866	.641	-.5	-.37	-.342	-.253	-.959	∓695
50	.45	.766	.345	.643	.289	.5	.225	-.866	∓39	-.939	-.422	-.342	∓154
60	.24	.866	.208	.5	.12	0	0	-1.0	∓24	-.866	-.208	.5	∓12
70	.128	.939	.12	.342	.044	∓5	-.064	-.866	∓111	-.179	.022	.984	.126
80	.054	.984	.053	.174	.009	∓866	-.046	-.5	∓027	-.693	.035	.766	.042
90	0	1.0	0	0	0	-1.0	0	0	0	1.0	0	0	0
100	-.0064	.984	∓006	-.174	.001	∓866	.006	.5	∓003	.643	∓004	-.766	.005
110	-.011	.939	∓011	-.342	.004	∓5	.005	.866	∓004	-.174	±002	-.984	.011
120	-.015	.866	∓013	-.5	.007	0	0	1.0	∓015	-.866	.013	-.5	.007
130	-.025	.766	∓019	-.643	.016	.5	-.012	.866	∓022	-.939	.023	.342	∓008
140	-.043	.643	∓028	-.766	.033	.866	-.038	.5	∓022	-.342	.015	.939	∓041
150	-.072	.5	∓036	-.866	.062	1.0	-.078	0	0	.5	∓036	.866	∓062
160	-.098	.342	∓035	-.94	.092	.866	-.085	-.5	.049	.984	∓096	-.174	∓017
170	-.165	.174	∓029	-.984	.165	.5	.084	-.866	.145	.766	∓129	-.643	±108
180	-.245	0	0	-1.0	.245	0	0	-1.0	.245	0	0	-1.0	.246
Average			.1008		.195		.124		-.006		.02		∓058
2 x Av.			.2016		.390		.248		-.012		.04		∓116

Analyzing E. M. F. Curve.

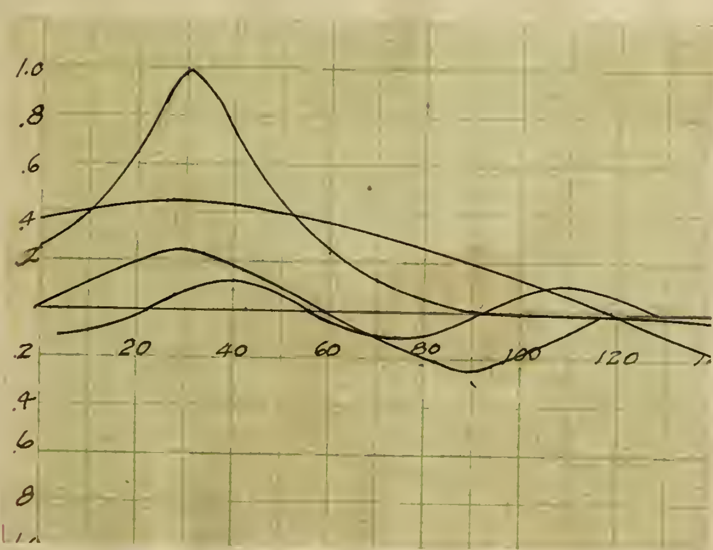
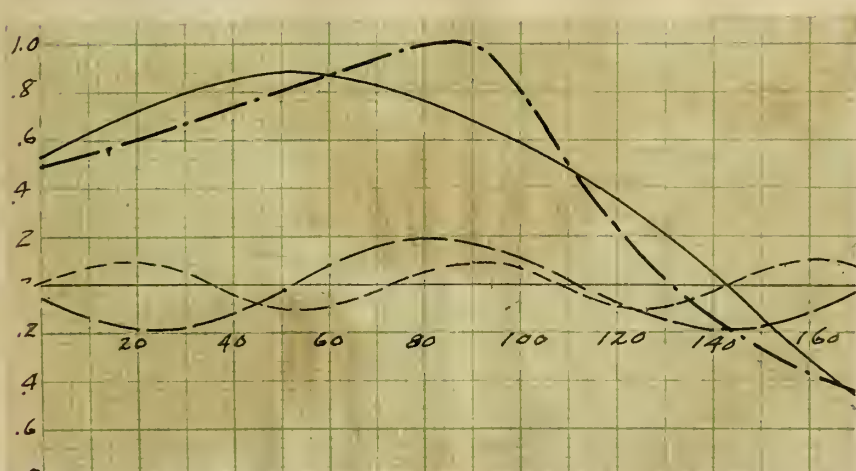


TABLE XV

Angle	F	Sin θ	F sin θ	Cos θ	F cos θ	Sin 3θ	F sin 3θ	Cos 3θ	F cos 3θ	Sin 5θ	F sin 5θ	Cos 5θ	F cos 5θ
0	.5	0	0	1.0	.5	0	0	1.0	.5	0	0	1	.5
10	.55	.174	.0958	.984	.541	.5	.275	.866	.476	.766	.422	.643	.354
20	.60	.342	.205	.94	.564	.866	.52	.5	.30	.984	.59	-.174	-.104
30	.67	.5	.335	.866	.58	1.0	.67	0	0	.5	.335	-.866	-.58
40	.73	.643	.47	.766	.56	.866	.632	-.5	-.365	-.342	-.25	-.939	-.686
50	.80	.766	.613	.693	.515	.5	.40	-.866	-.693	-.939	-.754	-.342	-.274
60	.87	.866	.754	.5	.435	0	.0	-1.0	-.87	-.866	-.754	-.5	.435
70	.94	.939	.882	.342	.322	-.5	-.47	-.866	-.815	-.174	-.164	.984	.924
80	.99	.984	.974	.174	.172	-.866	-.857	-.5	-.495	.643	.637	.766	.758
90	1.0	1.0	1.0	0	0	-1.0	-1.0	0	0	1.0	1.0	0	0
100	.80	.984	.786	-.174	-.139	-.866	-.693	.5	.40	.643	.515	-.766	-.614
110	.50	.939	.469	-.342	-.171	-.5	-.25	.866	.433	-.174	-.087	-.984	-.492
120	.23	.866	.199	-.5	-.115	0	0	1.0	.23	-.866	-.199	-.5	-.115
130	.03	.766	.023	-.643	-.0193	.5	.015	.866	.026	-.939	-.028	.342	.01
140	-.14	.643	-.09	-.766	.106	.866	-.121	.5	-.07	-.342	.48	.94	-.131
150	-.26	.5	-.13	-.866	.225	1.0	-.26	0	0	.5	-.13	.866	-.063
160	-.36	.342	-.123	-.94	.338	.866	-.312	-.5	.18	.984	-.354	.174	-.063
170	-.44	.174	-.077	-.984	.432	.5	-.22	-.866	.382	.766	-.338	-.643	.284
180	-.5	0	0	-1.0	.5	0	0	-1.0	.5	0	0	-1.0	5
Average			.355		.269		-.093		-.021	.051			-.001
2 x Av.			.710		.538		-.186		-.042	.102			-.002



VII

POLYPHASE CIRCUITS AND POWER

The alternating current generator is built in two general types, that is, rotating field and stationary field. In these two general types the windings are placed symmetrically on the armature whether the machine is wound for the generation of single, two or three phase power.

When the alternator has a rotating field it is evident that the armature windings are stationary and hence no collector rings or brushes are required. With a stationary field both collector rings and brushes must be placed on the machine in order to collect the current. With the single phase machine two collector rings and two brushes are always used with a stationary field winding, while a 3-phase machine requires either 6 or 3 rings. This latter condition is made possible only when the windings are properly interconnected.

The armature windings of a 3-phase alternator are symmetrically spaced and the phase windings, number 1, 2, and 3, are placed 120 electrical degrees apart. It is, therefore, evident that the voltages generated in these three windings will be 120 electrical degrees apart and may be represented in time phase by the following waves. See figure 57.

From the above diagram it is seen that wave #1 is 120° in advance of wave #2 and 240° in advance of wave #3. Hence by the proper connection of the windings the following diagram is made

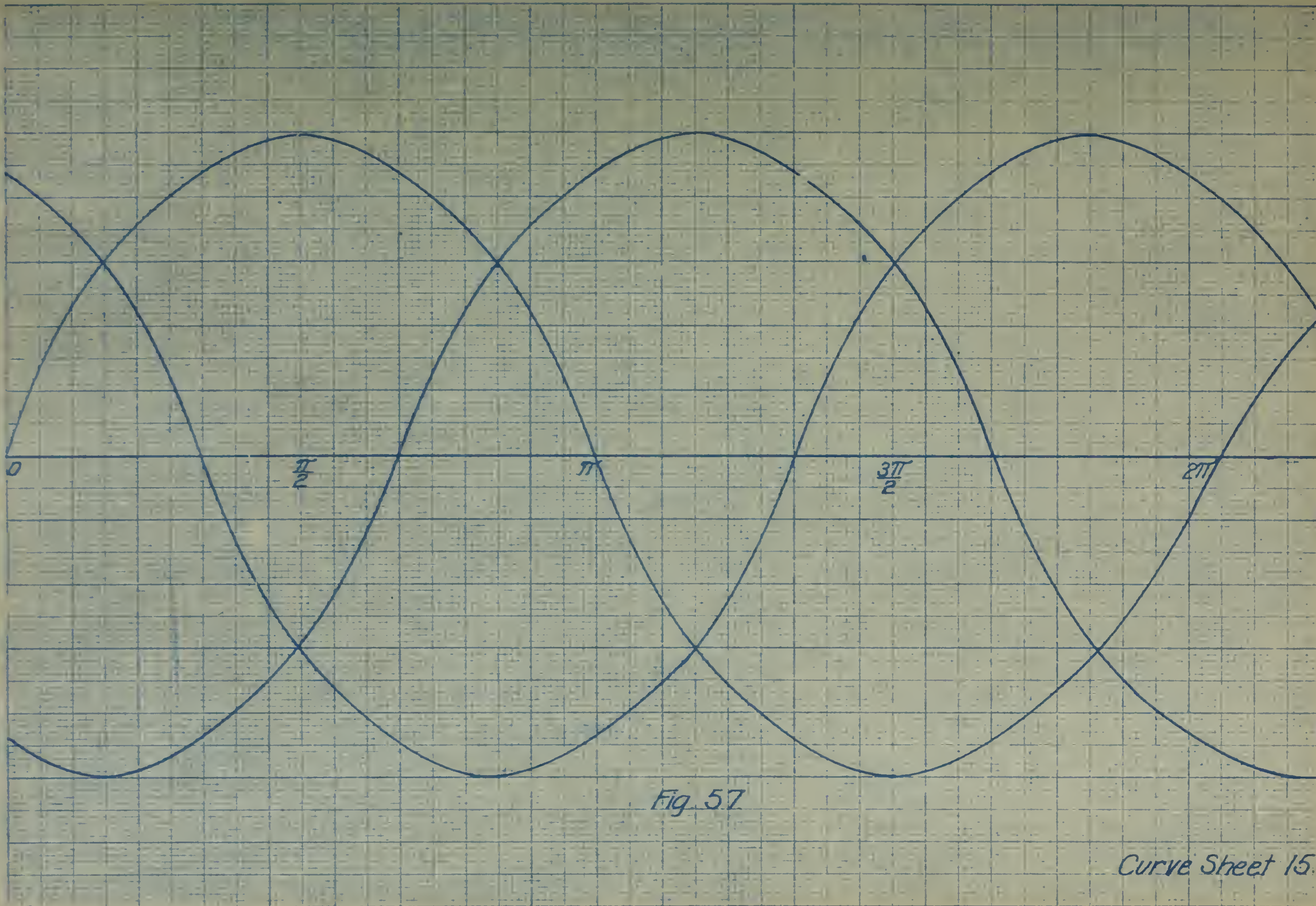


Fig. 57

Curve Sheet 15.

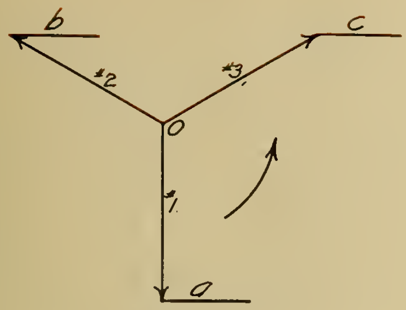


Fig. 59

possible. This diagram shows the vector a, 120° in advance of vector b, and 240° in advance of vector c. This is known as the "Wye" or "Star" connection of alternators. The three lines are then connected to rings at a, b, and c, and the voltage between lines is found to be the vector difference of the winding voltages oa and oc, or oa and ob, or ob and oc as represented in the following diagram. Voltage between lines a and c is,

$oa + (-oc) = E_1$. In the same manner the other line voltage may be found since $E_2 = oc + (-ob)$ and $E_3 = ob + (-oa)$. From the diagram it is seen that E_1, E_2, and E_3 are displaced 120 electrical degrees the same as the voltages of the windings.

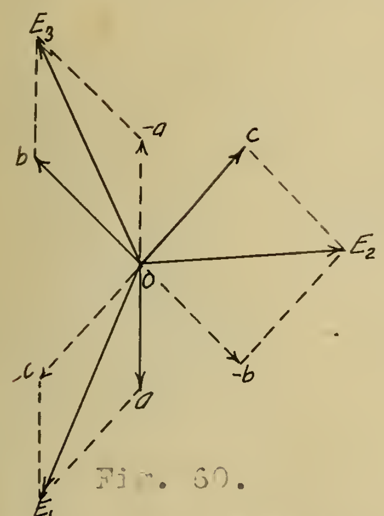


Fig. 60.

The windings of an alternator may also be so connected as to form a closed circuit which may be represented by a delta, as shown below.

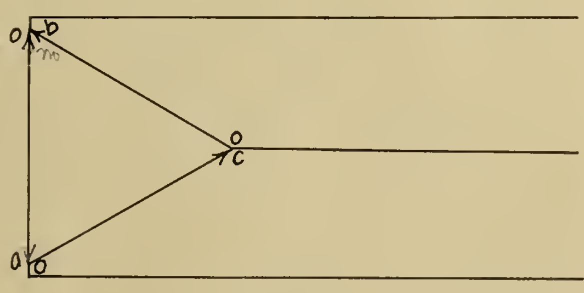


Fig. 61.

In this case oa, ob and oc represent the voltages generated in the windings and the line voltage as measured across any two lines will be in phase and equal to the voltage of the winding connected to the line under discussion.

The ratio of the line voltage to the winding voltage for, y, connected armatures (assuming sine waves) is $\sqrt{3}$ to 1 as is easily proven from the diagram on page 109. The ratio of the line current to the winding current when the windings are $\triangle$ connected is $\sqrt{3}$ to 1, assuming balanced load and sine waves of current.

Since power is the product of voltage and current, it is evident that for a single phase machine there will be at least two pulsations of power output for each cycle.

Let $e = E \sin \theta$

$i = I \sin \theta$, which is assuming the current is in phase with the voltage.

Power $= ei = E\,I \sin^2 \theta$

but,

$$\sin^2 \theta = \frac{1}{2} - \frac{1}{2} \cos 2\theta$$

or,

$P = ei = E\,I\left(\frac{1}{2} - \frac{1}{2} \cos 2\theta\right)$ which is a quantity varying at twice the frequency of the voltage or current.

To show that the power output of a polyphase generator is constant. Take the case of a two phase alternator and let e_1 = instantaneous value of voltage of one phase, i = instantaneous value of current from the same phase, and let e_2 and i_2 be the corresponding values for the second phase. Let the load be

balanced between the two phase and let θ be the power factor. Then the output of phase one is,

$$P_1 = e_1 \, i_1 = E_1 \, I_1 \sin \alpha \, \sin (\alpha - \theta)$$

$$P_2 = e_2 \, i_2 = E_2 \, I_2 \cos \alpha \, \cos (\alpha - \theta)$$

$$P = P_1 + P_2 = e_1 \, i_1 + e_2 \, i_2 = E_1 \, I_1 \sin \alpha \sin (\alpha - \theta) + E_2 \, I_2 \cos \alpha \cos (\alpha - \theta)$$

But, $E_1 \, I_1 + E_2 \, I_2 = E \, I$, and the above equation reduces to,

$P = 2E \, I \cos \theta$ which is constant since θ was assumed constant.

In like manner the power output, with balanced load, of a three phase generator may be proven to be $P = \sqrt{3} \, E \, I \cos \theta$, where E is the voltage between lines, I is the current in all the lines and θ is the power factor angle. This, of course, assumes a balanced load and sine wave relations of current and voltage.

It has previously been shown that, for sine waves of voltages, the line voltage is $\sqrt{3}$ times the winding voltage in a "Wye" connected generator. But, since generators do not always produce a sine wave voltage, it is of interest to see what relation exists under extreme conditions of variation from this condition.

The following tables gives values that have been obtained from a three phase generator connected in "y". Where e_1 is the voltage generated by one phase and e_2 is the voltage generated by the second phase, the windings being 120° apart in space. "A" is the vector difference of the two waves and the ratio between the line voltage and the winding voltage has been found to be 1.51 to 1 and not 1.73 to 1 as would be the case if the e.m.f. was a pure sine wave in each coil. On curve sheet number 16 is shown the two waves plotted to scale in their proper phase position. Curve

TABLE XVI

θ	e_1	e_2	$e_1 - e_2$	e_1^2
270	0	-.072	.072	
280	.0064	-.098	.1044	
290	.011	-.165	.176	
300	.015	-.245	.26	
310	.025	-.39	.415	
320	.043	-.65	.693	
330	.072	-1.0	1.072	
340	.098	-.74	.818	
350	.165	-.45	.615	
0	.245	-.24	.485	.06002
10	.39	-.128	.518	.1521
20	.65	-.054	.704	.4225
30	1.0	0	1.00	1.00
40	.74	.0064	.7336	.5476
50	.45	.011	.439	.2025
60	.24	.015	.225	.0576
70	.128	.025	.103	.01638
80	.054	.043	.011	.00291
90	0	.072	-.072	0
100	-.0064	.098	-.1044	.00004
110	-.011	.165	-.176	.00012
120	-.015	.245	-.26	.00022
130	-.025	.39	-.415	.000625
140	-.043	.65	-.693	.001849
150	-.072	1.0	-1.072	.005184
160	-.098	.74	-.818	.0096
170	-.165	.45	-.615	.02622
180	-.245	.24	-.485	
190	-.39	.128	-.518	
200	-.65	.054	-.704	
210	-1.0	0	-1.0	
220	-.74	-.0064	-.7336	
230	-.45	-.011	-.439	
240	-.24	-.015	-.225	
250	-.128	-.025	-.103	
260	-.054	-.043	-.011	

Mean $e_1^2 = .3735$

Mean $(e_1 - e_2)^2 = .564$

Ratio $= \frac{.564}{.3735} = 1.51$

TABLE VXII

θ	e_1	e_2	$e_1 - e_2$	e_1^2	$(e_1 - e_2)^2$
0	.245	-.24	.485	.06002	.2352
10	.39	-.128	.518	.1521	.2683
20	.65	-.054	.704	.4225	.4956
30	1.0	0	1.00	1.00	1.000
40	.74	.0064	.7336	.5476	.5373
50	.45	.011	.439	.2025	.1927
60	.24	.015	.225	.0576	.0506
70	.128	.025	.103	.01638	.0106
80	.054	.043	.011	.00291	.0001
90	0	.072	-.072	0	0
100	-.0064	.098	-.1044	.00004	.0109
110	-.011	.165	-.176	.00012	.031
120	-.015	.245	-.26	.00022	.0676
130	-.025	.39	-.415	.000625	.1722
140	-.043	.65	-.693	.001849	.480
150	-.072	1.0	-1.072	.005184	1.149
160	-.098	.74	-.818	.0096	.6694
170	-.165	.45	-.615	.02622	.3788

Curve Sheet 16

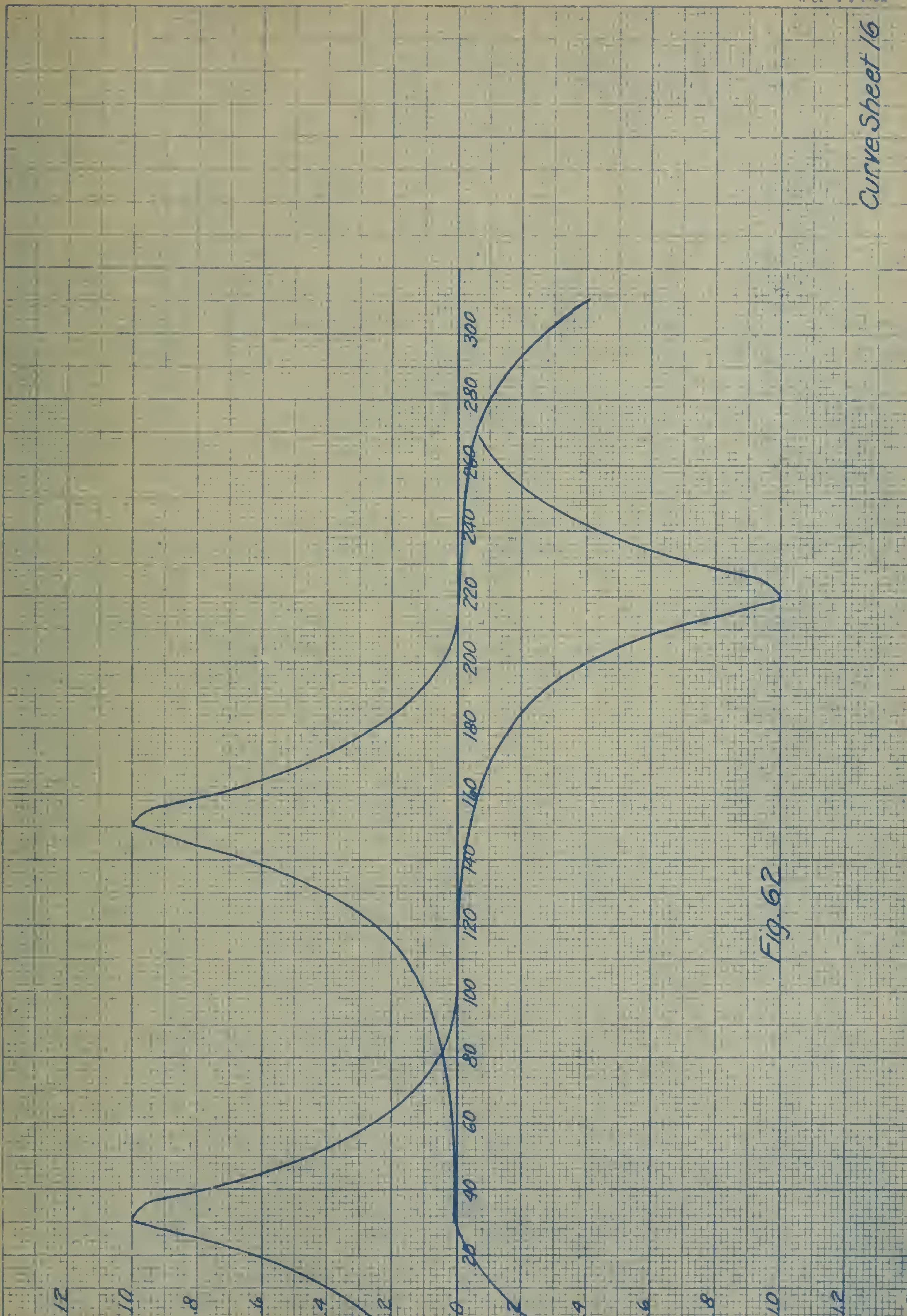


Fig. 62

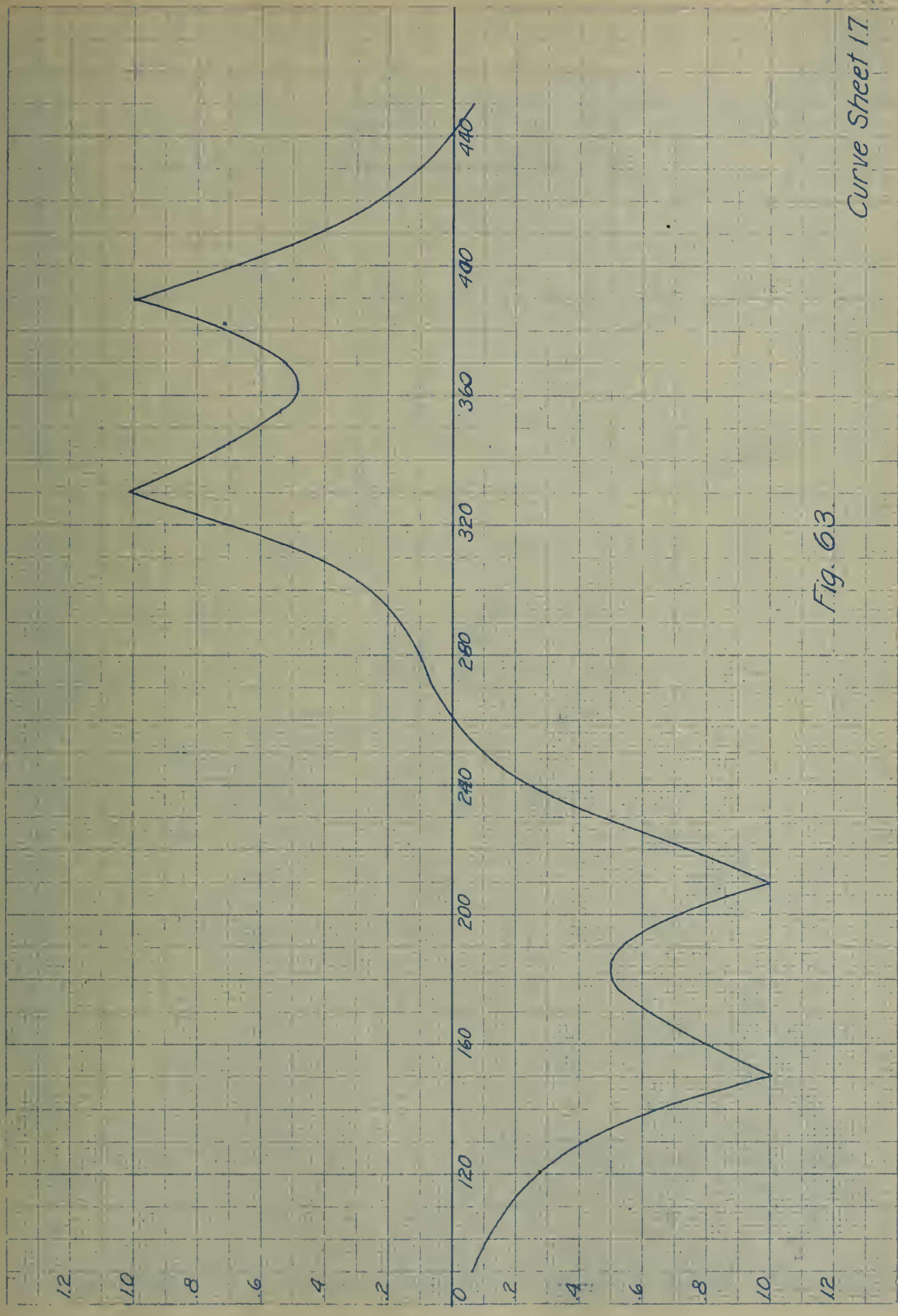
sheet 17 shows the resultant wave obtained by taking the vector difference of the above waves.

If the wave plotted on curve sheet 17 is carefully analyzed for a triple harmonic it is found that no such harmonic exists. While each of the component curves has a very pronounced triple harmonic. The above leads to the conclusion that the voltage between lines of a "Wye" connected generator can not contain a triple harmonic. This is also made very plain by drawing the three fundamental waves 120 electrical degrees apart and then drawing in the triple for each wave. By this method it is seen that, since the line voltage is always the difference of potential of two waves, there can be no difference of potential between the triple waves as they are in phase with each other.

Again in a "Wye" connected generator the end of all three windings are connected to one common point, and since the triples are all in phase with each other it is evident that the free end of all three windings must be at the same potential. The potential difference between any of these ends will therefore be zero if only a triple harmonic exists.

Let i = the instantaneous value of current flowing in a line, and let $i = I_1 \cos \theta + I_3 \cos 3\theta + I_5 \cos 5\theta$. The effective value of the current will then be mean i^2, and

$$\text{mean } i^2 = \int_0^{2\pi} I_1^2 \frac{\cos^2 \theta}{2\pi} d\theta + \int_0^{2\pi} 2 I_1 I_3 \frac{\cos \theta}{2\pi} \cos 3\theta \, d\theta + \int_0^{2\pi} I_1 \frac{I_5 \cos \theta}{2\pi} \cos d\theta \, 5\theta + \int_0^{2\pi} I_3 \frac{\cos^2 3\theta}{2\pi} d\theta + \int_0^{2\pi} I_5 \frac{\cos^2 5\theta \, d\theta}{2\pi}$$



Curve Sheet 17
1.2
1.0
.8
.6
.4
.2
0
.2
.4
.6
.8
1.0
1.2
120
160
200
240
280
320
360
400
440

Fig. 63

$$\sqrt{\text{Mean } i^2} = \sqrt{\frac{I_1^2}{2} + \frac{I_3^2}{2} + \frac{I_5^2}{2}} = I_{eff.}$$

The fact that a three phase alternator can be operated with three wires is easily shown from the following considerations. Assume a "Wye" connected alternator with the neutral brought out as indicated below.

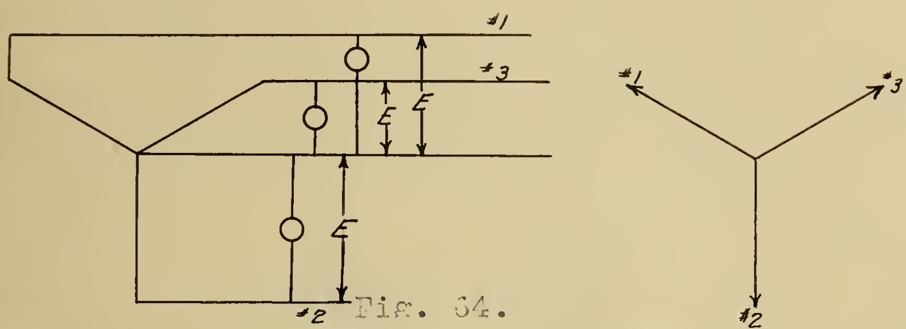


Fig. 34.

Assume balanced load on all three phases and connect all three phases to the neutral wire as indicated.

The current delivered by phase #1 is,

$$i = I \sin\alpha$$

Phase #2,

$$i_2 = I_2 \sin(\alpha + 120)$$

Phase #3,

$$i_3 = I_3 \sin(\alpha + 240^\circ)$$

and the current in the neutral must, therefore, be the sum of these three currents, i neutral $= i_1 + i_2 + i_3 = I_1 \sin + I_2 \sin(\alpha + 120) + I_3 \sin(\alpha + 240)$, and since balanced load was assumed, i neutral $= I(\sin\alpha + \sin(\alpha + 120) + \sin(\alpha + 240))$, which reduces to zero and, therefore, the neutral current is zero. Since, the above shows that for balanced load, the neutral current is zero it is evident that no neutral wire need be used and the three line wires are all that are necessary, the load being connected between the outside lines.

VIII

POWER MEASUREMENTS

Alternating current power is usually measured by means of a wattmeter or a combination of wattmeters. The essential parts of the wattmeter are; (a) current coil which is fixed in position and through which the load current must pass, (b) voltage coil, which is movable and so connected that the current through this coil is proportional to the voltage impressed on the load, as shown in figure 65.

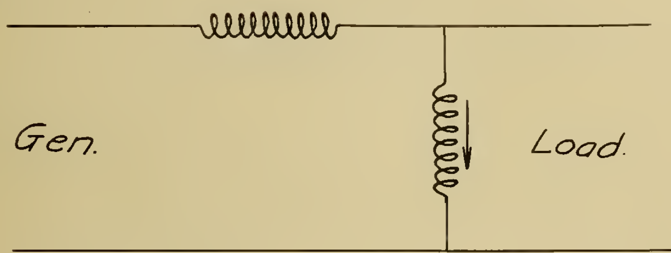


Fig. 65.

The movable or voltage coil of the commercial wattmeter is usually placed inside the current coil and has its axis so placed that the flux produced by the current coil will be at right angles to the flux produced by the voltage coil when the later is in the zero position.

Since the torque acting on the movable coil is proportional to the product of the flux produced by the current and voltage coil, it is evident that this torque will also be proportional to the product of the currents in these coils provided no iron is introduced in the magnetic circuit.

Now let $i = I \sin (\alpha - \theta)$ = any sine wave of current flowing in the current coil and let $e = E \sin \theta$ be the e.m.f. impressed

on the coil. The field produced by the current coil will be $K\, I \sin(\alpha - \theta)$ where K is a constant depending on the construction of the coil. The current in the potential coil will be $\frac{E \sin\alpha}{Z}$, where Z is the impedance of the voltage coil. Now if Z is not so made that r is very great in comparison to x, it is evident that the current in the voltage coil will not be in phase with $E \sin\alpha$ and will not produce a magnetic field proportional at each instant to $E \sin\alpha$. However, the commercial wattmeter x is made very small and r very large so that the above error is very small indeed, and it is safe to say,

$$i \text{ in voltage coil} = \frac{E \sin\alpha}{r} .$$

The torque is then,

$$K \sin(\alpha - \theta)\, K \frac{E \sin\alpha}{r}$$

$$= \frac{K' E I}{r} \sin\alpha \sin(\alpha - \theta)$$

$$= \frac{K' E I}{r} \sin^2\alpha \cos\theta - \sin\alpha \cos\alpha \sin\theta.$$

Now if the voltage coil is held so that its magnetic field is always at right angles to the magnetic field of the current coil, the average torque is,

$$\frac{1}{2\pi}\int_0^{2\pi} \frac{K' E I}{r} \sin^2\alpha \cos\theta - \sin\alpha \cos\alpha \sin\theta$$

$$= \frac{K' EI}{2\, r} \cos\theta$$

$$= K'' \frac{EI}{2} \cos\theta = \text{average torque},$$

When E is the effective value of voltage, and I is the effective value of current, then the average torque indicated by a wattmeter of this type is $K'' E I \cos\theta$, and it is seen that the commercial

wattmeter can be so calibrated that its indication is equal to E I cos θ, where E is the effective voltage, I the effective current, and θ is the angle of lag or lead.

Power Measurements of Three Phase Circuits.

It has already been shown that the power output of a three phase generator with balanced load and sine wave current and voltage is $P = E\,I\,\sqrt{3}$, where E is the effective line voltage, and I is the effective line current. P, of course, is the total power output in watts.

In the following diagram assume that the positive direction of flow of current is from the neutral as indicated by the vectors. Let e_1, e_2 and e_3 represent the instantaneous values

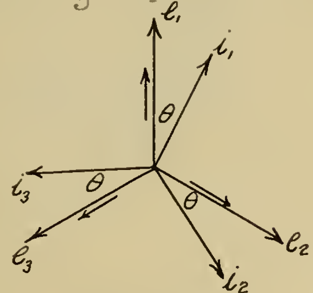


Fig. 66.

of voltages. Let i_1, i_2 and i_3 represent the instantaneous values of current and let $\cos\theta$ = the power factor. Now according to Kirchoff's Law the current leaving a junction must equal the current flowing into the junction, or $i_1 + i_2 + i_3 = 0$. From which it is seen $-i_2 = i_1 + i_3$, and since the total power of any three phase circuit is $P = e_1 i_1 + e_2 i_2 + e_3 i_3$ by substitution $= e_1 i_1 + e_2 (-i_1 - i_3) + e_3 i_3 = i_1 (e_1 - e_2) + i_3 (e_3 - e_2)$, which shows that it is only necessary to use two wattmeters connected as in figure 67 .

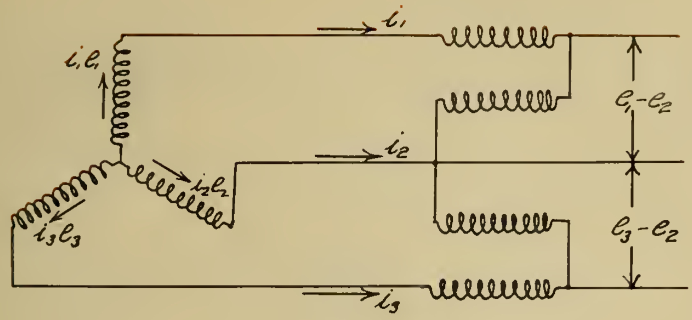


Fig. 67.

The vector diagram for the above wattmeter connection (assuming balanced load and sine wave relations) is shown in fig. 68.

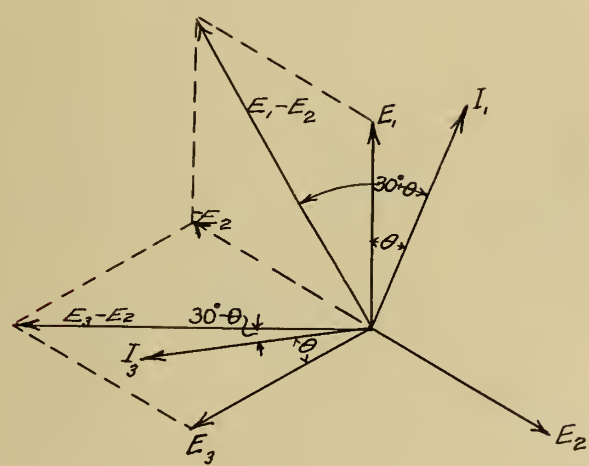


Fig. 68.

Wattmeter #1 reads, $i_1 (e_1 - e_2)$

Wattmeter #1 reads, $E_1 I_1 \cos (30^0 + \theta)$

Wattmeter #2 reads, $i_3 (e_3 - e_2)$

Wattmeter #2 reads, $E_3 I_3 \cos (30^0 - \theta)$

Total power is $i_1 (e_1 - e_3) + i_3 (e_3 - e_2)$

Total power is $E_1 I_1 \cos (30^0 + \theta) + E_3 I_3 \cos (30^0 - \theta)$

The above equations show that two wattmeters properly connected are

sufficient for the measurement of three phase power.

$$\text{Let } E_{1\text{ eff.}} = E_{2\text{ eff.}} = E_{3\text{ eff.}} = 1.00$$

$$I_{1\text{ eff.}} = I_{2\text{eff.}} = I_{3\text{ eff.}} = 1.00$$

Find power output for θ = zero to $\theta = 90^o$.

TABLE XVIII

θ	$\cos(30^o + \theta)$	$\cos(30^o - \theta)$	W_1	W_2	$W_1 + W_3$
0	.866	.866	.866	.866	1.732
10	.766	.939	.766	.939	1.705
20	.642	.984	.642	.984	1.626
30	.500	1.000	.500	1.000	1.500
40	.342	.984	.342	.984	1.326
50	.173	.939	.173	.939	1.112
60	.000	.866	.000	.866	.866
70	-.173	.766	-.173	.766	.593
80	-.342	.642	-.342	.642	.300
90	-.500	.500	-.500	.500	000

The curve on page 123shows these data plotted to scale, from which is seen the variation of output of a three phase generator with varying power factor but constant potential and constant current.

EXAMPLE

A three phase, 100 volt circuit has resistances 6 ohms, 8 ohms, and 8 ohms connected between the line wires. Find the readings of the two wattmeters (W_1 and W_2) when connected as shown below.

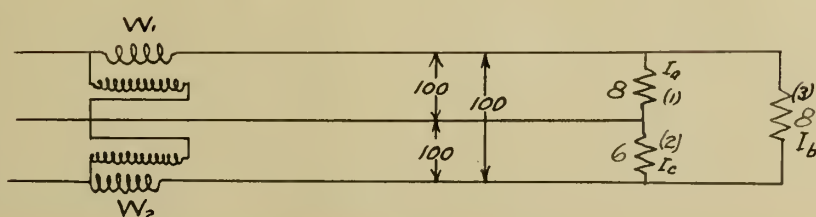


Fig. 70.

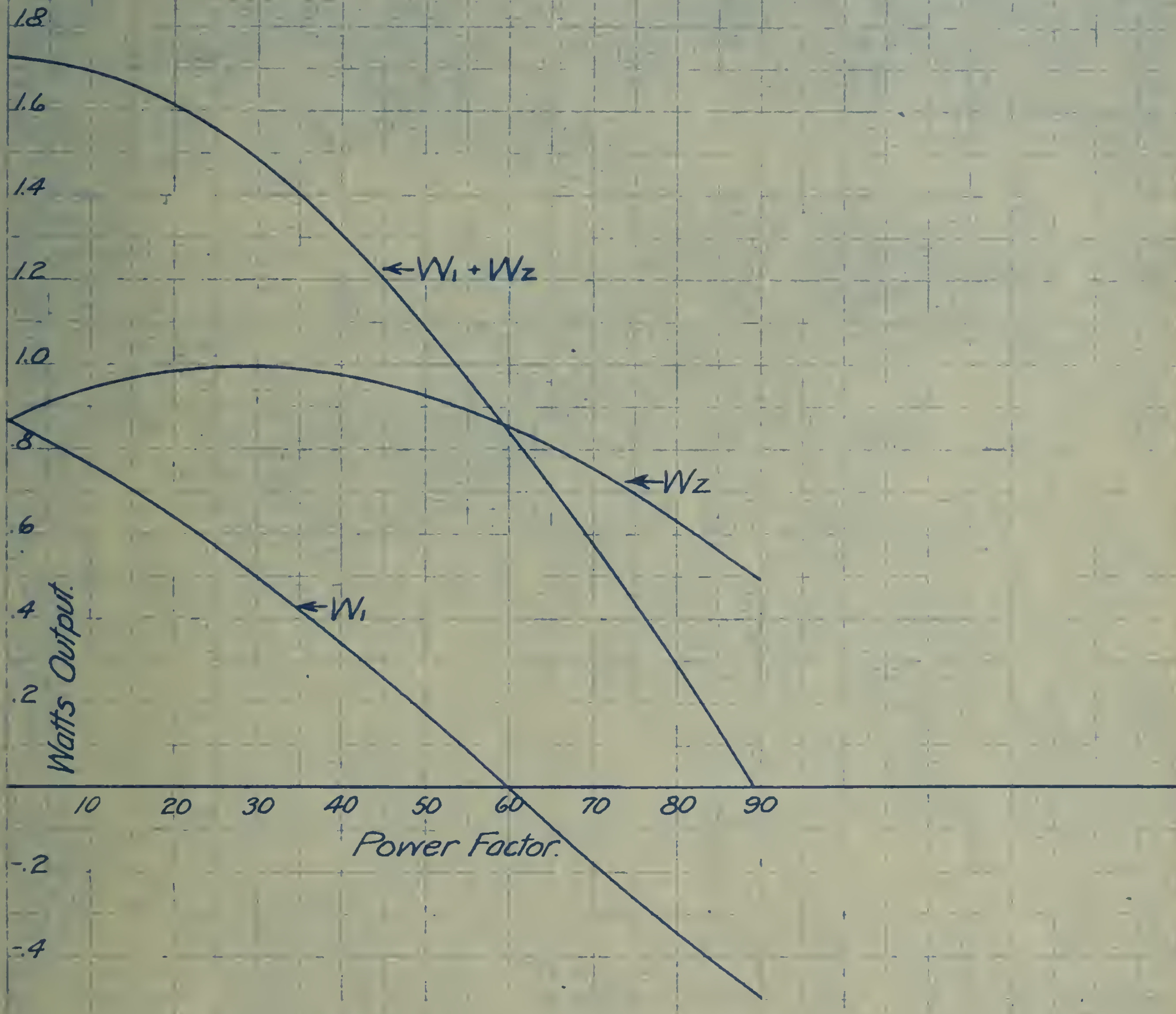


Fig. 69.

Curve Sheet 18.

Current in branch (1) is $\frac{100}{8}$ = 12.5 amperes.

Current in branch (2) is $\frac{100}{6}$ = 16.66 amperes.

Current in branch (3) is $\frac{100}{8}$ = 12.5 amperes.

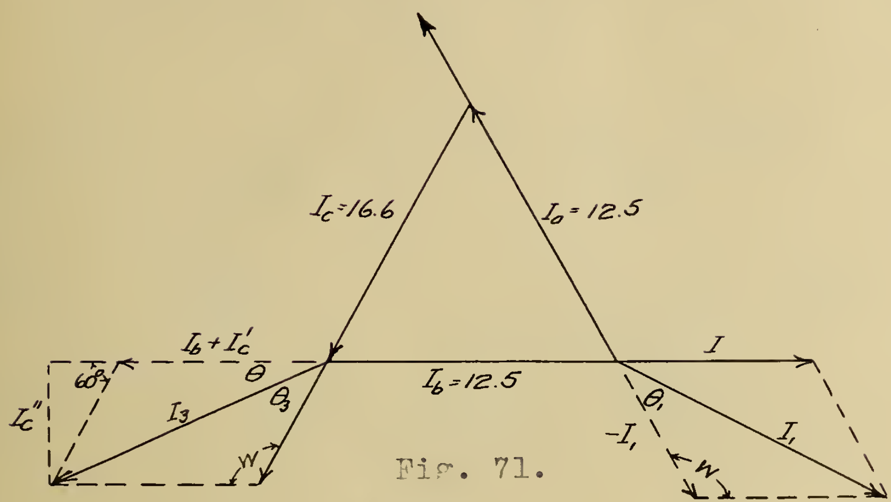


Fig. 71.

$$I_1 = I_b + (-I_a) = 21.7$$

$$I_3 = I_c + (-I_b) = 25.35$$

From the construction of the figure it is evident that angle ω = 120° and,

$$I_3 = I_c^2 + I_b^2 - 2\ I_c\ I_b \cos 120^\circ = 25.35$$

$$I_1 = I_a^2 + I_b^2 - 2\ I_a\ I_b \cos 120^\circ = 21.7$$

By construction $\theta_1 = 30^\circ$.

$$I_b + I'_c = 12.5 + 16.6 \sin 60 = 20.8$$

$$I''_c = 16.6 \quad \cos \quad 60 \qquad = 14.38$$

$$\text{Tan } \theta = \frac{14.34}{20.8} = .692 \qquad \theta = 34^\circ\ 41'.$$

$$\theta_3 = 60^\circ - 34^\circ\ 41' = 25^\circ\ 19'.$$

$$\text{Cos } \theta_3 = .9039.$$

$$W_1 = E\ I_1 \cos \theta_1 = 100 \times 21.7 \times \cos 30^\circ = 1878 \text{ watts.}$$

$$W_3 = E\ I_3 \cos \theta_3 = 100 \times 25.35 \times \cos 25^\circ\ 19' = 2288 \text{ watts.}$$

Total power = 1878 + 2288 = 4166 watts.

Solution of the same problem by complex quantities.

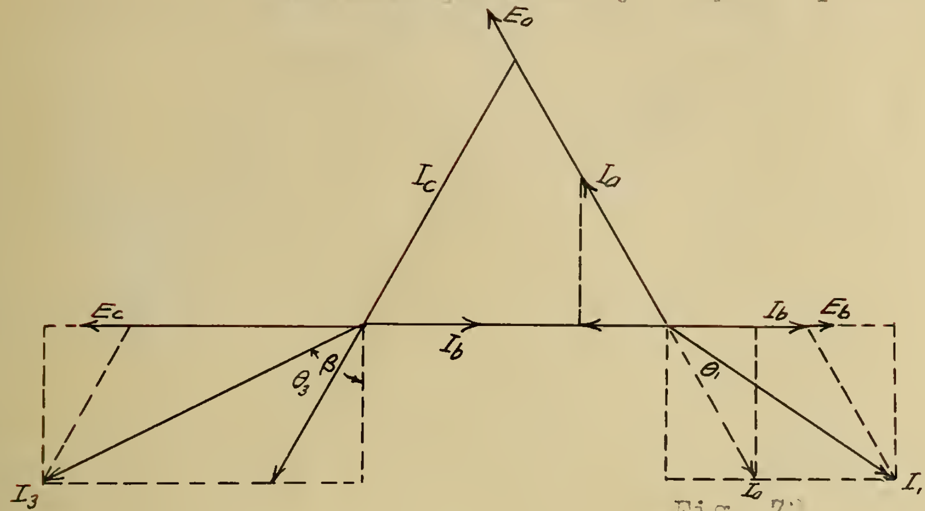


Fig. 72.

$I_a = I_a \cos 120^\circ + j\, I_a \sin 120^\circ$ but $-I$ is I rotated through 180° which can be done by multiplying by j^2.

$$-I_a = j^2\, I_a \cos 120^\circ + j^3\, I_a \sin 120^\circ$$

$$= -I_a \cos 120^\circ - j\, I_a \sin 120^\circ$$

$$I_1 = I_b + I_a \cos 120^\circ - j\, I_a \sin 120^\circ$$

$$= I_b + .5\, I_a - j\, .866\, I_a$$

$$I_1 = 12.5 + 6.25 - j\, 10.82 = 21.7 \text{ amp.}$$

$$\tan \theta_1 = \frac{10.82}{18.75} \qquad \theta_1 = 30^\circ \qquad \cos \theta_1 = .866$$

Again,

$$I_c = I_c \cos 240 - j\, I_c \sin 240$$

$$= -.5\, I_c + .866\, j\, I_c \qquad I_3 = -I_b + I_c$$

$$I_3 = -12.5 - 8.33 + j\, 14.44 = 25.34$$

$$\tan \beta_3 = \frac{20.83}{14.44} = 1.443 \qquad \beta_3 = 55^\circ\ 17'$$

$$\theta_3 = \beta_3 - 30^\circ = 25^\circ\ 17' \qquad \cos \theta_3 = .904.$$

$W_1 = 100 \times 21.7 \times .866 = 1878$ watts.

$W_2 = 100 \times 25.34 \times .904 = 2288$ watts.

$W_1 + W_2 = 4166$

Check, $E_a I_a = 1250$ $E_b I_b = 1250$ $E_c I_c = 1666$

$E_a I_a + E_b I_b + E_c I_c = 4166$ watts.

Either method may be used to solve problems of this general character although the latter method seems to be the least complicated. In any case the power factor of the load need not be unity in order to lead to a simple solution by these methods.

Problem. Given three resistances connected in Y as illustrated in figure 73. Find the current in each resistance and the power consumed by the resistances.

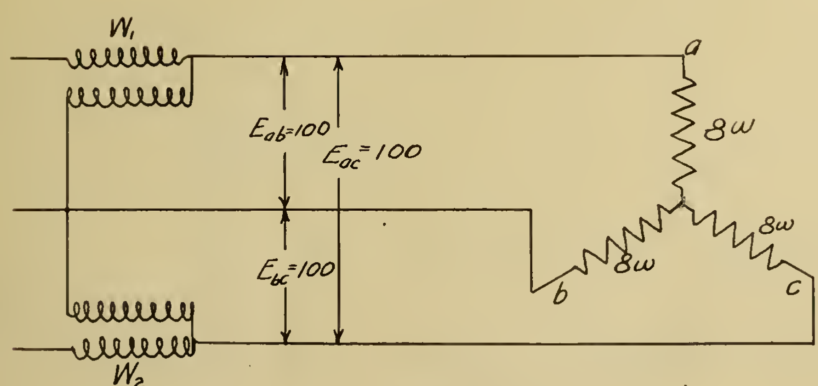


Fig. 73.

Using the diagram as illustrated in figure 74 the following equations may readily be developed.

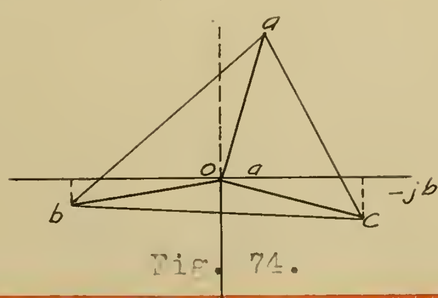


Fig. 74.

$$E\ (oc) = a - j\ b$$

$$E\ (ob) = -(100 - a) - j\ b$$

$$E\ (oa) = a - 50 + j\ (.866 - b)$$

$$I_{(oc)} = \frac{a - j\ b}{8} = \frac{a}{8} - j\ \frac{b}{8}$$

$$I_{(ob)} = \frac{-(100 - a) - j\ b}{8} = \frac{a}{8} - \frac{100}{8} - j\ \frac{b}{8}$$

$$I_{(oa)} = \frac{a - 50 + j\ (86.6 - b)}{6} = \frac{a}{6} - \frac{50}{6} + j\ \frac{86.6}{6} - j\ \frac{b}{6}$$

Since the algebraic sum of the currents meeting at a junction must always reduce to zero, it is evident that the algebraic sum of the horizontal and vertical components of these currents must reduce to zero.

Hence, $$\frac{a}{8} + \frac{a}{8} - \frac{100}{8} + \frac{a}{6} - \frac{50}{6} = 0$$

and, $$-\frac{b}{8} - \frac{b}{8} - \frac{b}{6} + \frac{86.6}{6} = 0$$

Whence, $a = 50$ and $b = 34.64$.

$$I_{(oc)} = \frac{50}{8} - j\ \frac{34.64}{8} = 7.59 \text{ amperes.}$$

$$I_{(ob)} = \frac{50}{8} - \frac{100}{8} - j\ \frac{34.64}{8} = 7.59 \text{ amperes.}$$

$$I_{(oa)} = \frac{50}{6} - \frac{50}{6} + j\ \frac{86.6}{6} = j\ \frac{34.64}{6} = 8.33 \text{ amperes.}$$

Redrawing the diagram to conform to the new values of currents which have been determined we have figure 75, which locates the neutral point.

$$I_{(oc)} = 6.25 - j\ 4.33$$

$$I_{(ob)} = -6.25 - j\ 4.33$$

$$I_{(oa)} = 0 + j\ 8.33.$$

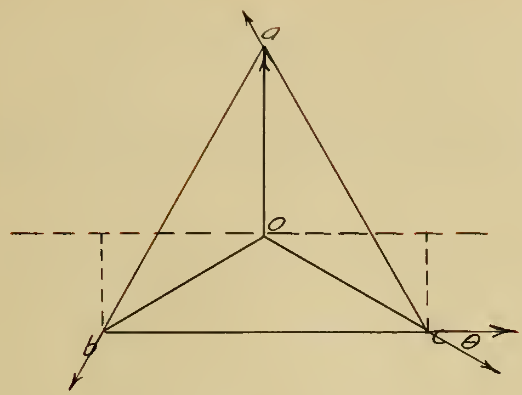


Fig. 75.

The voltage $E_{(oc)} = 8 \times 7.59 = 60.7$ volts.

The voltage $E_{(ob)} = 8 \times 7.59 = 60.7$ volts.

The voltage $E_{(oa)} = 6 \times 8.33 = 50.0$ volts.

Power consumed in branch,

$oc = 60.7 \times 7.59 = 461$ watts.

$ob = 60.7 \times 7.59 = 461$ watts.

$oa = 50 \quad \times 8.33 = 416.5$ watts,

and the total power consumed is,

$2 \times 461 + 416.5 = 1338.5$ watts.

Wattmeter W_1 is actuated by the current in oa and the voltage $E_{(a\ b)}$.

$W_1 = E_{ab} \times 8.33 \times \cos 30^\circ = 721$ watts.

$W_2 = E_{bc} \times 7.59 \times \cos \theta \quad = 625$ watts.

Total power $= W_1 + W_2 = 1346$ watts, which value checks closely with that obtained above.

IX

TWO PHASE - THREE PHASE TRANSFORMATION

It is often desirable to transform two phase power to three phase power and to accomplish this the "Scott System" was developed. This system consists of two single phase transformers having the following ratios.

Transformer A, E_p = 100 V. E_s = 100 V.

Transformer B, E_p = 100 V. E_s = 86.6 V.

The proper connection of these transformers is as shown below.

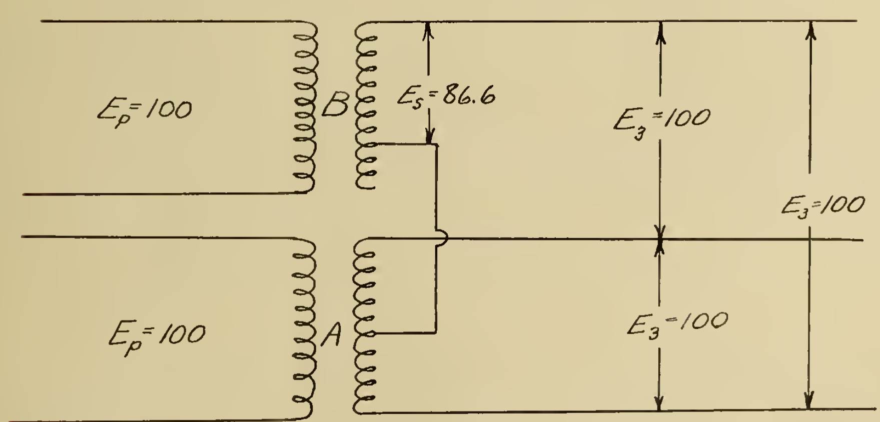


Fig. 76.

From the above figure it is evident that one side of the 86.6 volt secondary transformer A is connected to the middle point of the 100 volt secondary of transformer B. E_3 in each case represents the three phase voltage and is 100 volts, while E_p in each case represents the primary voltage which is impressed on each transformer and E_p impressed on transformer A is out of phase by 90 electrical degrees with E_p of transformer B.

Another scheme used to show the connections a little more clearly is shown below.

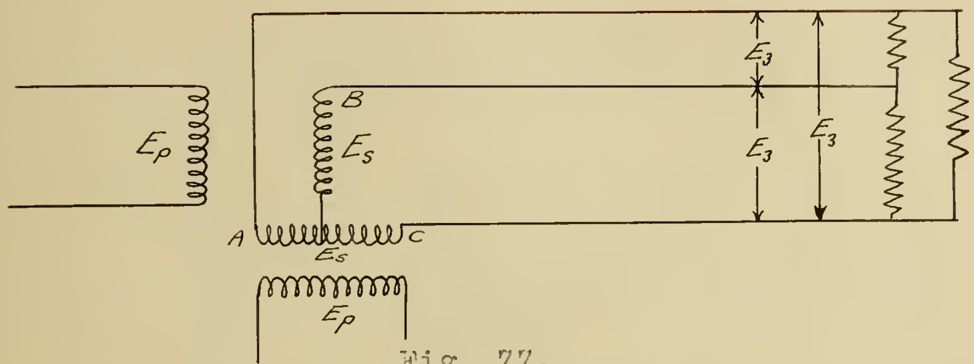


Fig. 77.

Now assume that E_3 in each case = 100 volts and assume I in the line = 100 amperes, the load being pure resistance.

The volt amperes of transformer A = 100 x 86.6 = 8660.

The volt amperes of transformer B = 100 x 100 = 10000,

and the total volt amperes will be 18660. The power in a three phase circuit is $E\,I\sqrt{3}$ and substituting the above values of E and I we have,

$$\text{Watts} = 1.73 \times 100 \times 100 = 17320,$$

while the volt amperes = 18660, from which it is evident there must be a power factor even with non-inductive load, and that the connection is not as effective as with the transformers operating single phase. The ratio of effectiveness being $\frac{17320}{18660} = .927$, or the output of the two transformers is reduced 7.3% by this connection.

Now let us investigate more closely the phase relation of currents and voltages in both the two phase and the three phase windings. From the above discussion it is evident that the volt-

age in winding ob is 90° out of phase with the voltage in oc, and 90° out of phase with the voltage in oa, since oa and oc are in phase with each other.

By applying Kirchoff's laws we may determine the phase position of the three voltages represented in the figure below, as E_{bc}, E_{ac} and E_{ab}.

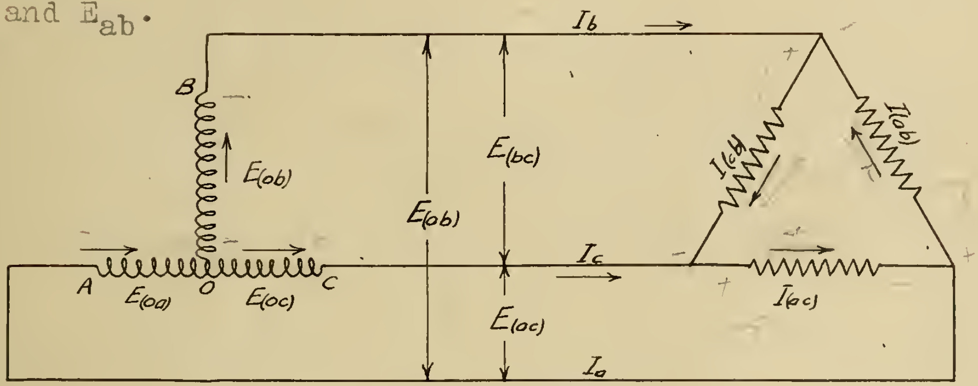


Fig. 78.

Assume that the counter clockwise direction of rotation is the direction of positive flow of current in any closed circuit of the net work and the positive direction of potential, Than,

$$(1)\quad E_{oc} + E_{bc} - E_{ab} = 0$$

$$E_{bc} = E_{ob} - E_{oc}$$

$$E_{ac} - E_{oa} - E_{oc} = 0$$

$$E_{ac} = E_{oa} - E_{oc}$$

$$E_{ab} + E_{oa} + E_{ob} = 0$$

$$E_{ab} = -E_{ob} - E_{oa}$$

Now assuming sine wave relations it is evident that

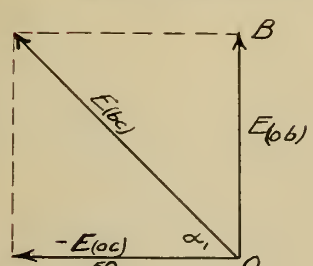


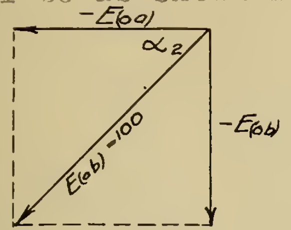
Fig. 79.

E_{bc} may be represented in phase position as in Fig. 79, and its effective value is,

$$\sqrt{50^2 + 86.6^2} = 100$$

Since $\cos\alpha_1 = .5 \quad \alpha_1 = 60^\circ$

Again E_{ab} may be represented in a vector diagram if sign waves of voltages are assumed and since $E_{ab} = -E_{ob} - E_{oa}$ the diagram will be as shown in Fig. 80, and its effective value will be



$$\sqrt{\overline{50}^2 + \overline{86.6}^2} = 100$$

Since $\cos \alpha_2 = .5 \quad \alpha_2 = 60^{\circ}$.

Fig. 80.

Now since E_{oa} and E_{oc} are in phase with each other and since $E_{ac} = E_{oa} + E_{oc}$, the vector diagram is represented by a straight line as in fig. 81,

and $E_{ac} = 50 + 50 = 100$.

Combining these vectors we have the following diagrams.

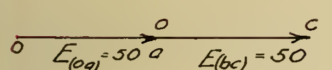


Fig. 81.

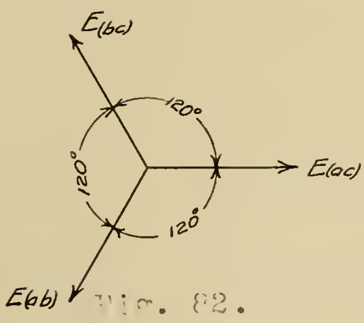


Fig. 82.

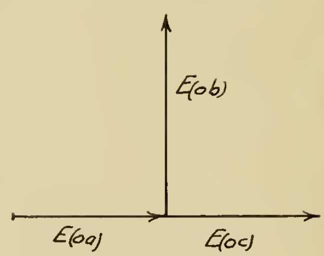


Fig. 83.

Tables XIX and XX show the calculated results for obtaining the instantaneous values of these waves and figure 84, curve sheet 19, show the waves plotted to scale and in their proper phase relation. In this table the maximum value of the three phase voltages is taken as unity.

TABLE XIX

θ	E_{ob}	$-E_{ob}$	$E_{oa}=E_{oc}$	E_{ob} $E_{ob}-E_{oc}$	$-E_{oa}$ or $-E_{oc}$	E_{ob} $-E_{oa}$ $-E_{ob}$	E_{ac} $E_{oa}+E_{oc}$ $oA + oC$
0	0	0	-.50	.50	.50	.5	-1.0
10	.150	-.150	-.492	.642	.492	.342	-.984
20	.296	-.296	-.469	.766	.469	.173	-.939
30	.433	-.433	-.433	.866	.433	0	-.866
40	.556	-.556	-.383	.939	.383	-.173	-.766
50	.663	-.663	-.321	.984	.321	-.342	-.642
60	.750	-.750	-.250	1.00	.250	-.50	-.500
70	.813	-.813	-.171	.984	.171	-.642	-.342
80	.853	-.853	-.086	.939	.086	-.766	-.173
90	.866	-.866	0	.866	0	-.866	0
100	.853	-.853	.086	.766	-.086	-.939	.173
110	.913	-.813	.171	.642	-.171	-.984	.342
120	.750	-.750	.250	.500	-.250	-1.00	.500
130	.663	-.663	.321	.342	-.321	-.984	.642
140	.556	-.556	.383	.173	-.383	-.939	.766
150	.433	-.433	.433	0	-.433	-.866	.866
160	.296	-.296	.469	-.173	-.469	-.766	.939
170	.150	-.150	.492	-.342	-.492	-.642	9.84
180	0	0	.50	-.50	-.50	-.500	1.0

TABLE XX

θ	E_{ob}	$-E_{ob}$	$E_{oa}=E_{oc}$	E_{ob} $E_{ob}-E_{oc}$	$-E_{oa}$ or $-E_{oc}$	E_{ob} $-E_{oa}-E_{ob}$	E_{ac} $E_{oa}+E_{oc}$
180	0	0	.5	-.50	-.50	-.5	1.00
190	-.150	.150	.492	-.642	-.492	-.342	.984
200	-.296	.296	.469	-.766	-.469	-.173	.939
210	-.433	.433	.433	-.866	-.433	0	.866
220	-.556	.556	.383	-.939	-.383	.173	.766
230	-.663	.663	.321	-.984	-.321	.342	.642
240	-.750	.750	.250	-1.00	-.250	.50	.50
250	-.813	.813	.171	-.984	-.171	.642	.342
260	-.853	.853	.086	-.939	-.086	.766	.173
270	-.866	.866	0	-.866	0	.866	0
280	-.853	.853	-.086	-.766	.086	.939	.173
290	-.813	.813	-.171	-.642	.171	.984	.342
300	-.750	.750	-.250	-.500	.250	1.00	.50
310	-.663	.663	-.321	-.342	.321	.984	.642
320	-.556	.556	-.383	-.173	.383	.939	.766
330	-.433	.433	-.433	0	.433	.866	.866
340	-.296	.296	-.469	.173	.469	.766	.939
350	-.150	.150	-.492	.342	.492	.642	.984
360	0	0	-.5	.50	.50	.50	1.00

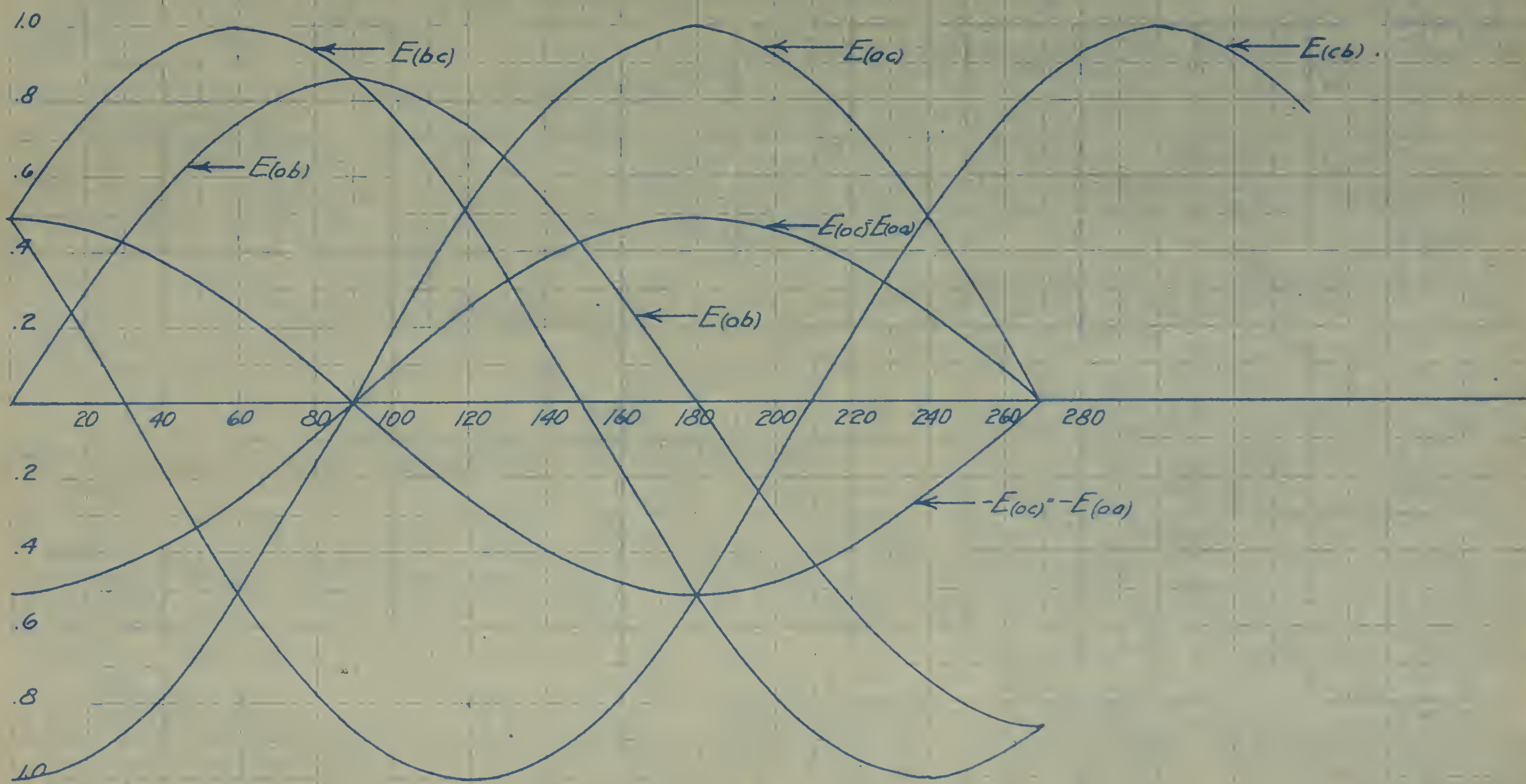


Fig. 84.

Curve Sheet 19.

Current Relations

It has previously been shown that the currents in the three phase lines are equal for balanced load. It would then naturally follow, since, transformer A has 86.6% as many turns in its secondary winding as transformer B that there would be 13.3% more ampere turns on transformer B both primary and secondary as there would be on transformer A both primary and secondary and hence the load would not be evenly distributed between the two. This, however, is not the case as will be shown. By application of Kirchoff's Law, (i.e.) "The current entering a junction must be equal to the current leaving the junction", we may determine the phase position of the various currents with respect to any voltage that is applied to the net work.

Referring to fig. 78, let us assume that the positive direction of flow of current is away from the junction and that the current entering the junction is negative. Then it is evident that,

$$(1)\quad -I_{ac} + I_{ab} + I_a = 0$$

$$(2)\quad I_{cb} - I_{ab} - I_b = 0$$

$$(3)\quad -I_c + I_{ac} - I_{cb} = 0$$

$$I_a = I_{ac} - I_{ab}$$

$$I_b = I_{cb} - I_{ab}$$

$$I_c = I_{ac} - I_{cb}$$

Now we know that the current I_{cb} is in phase with the voltage E_{cb}; I_{ac} is also in phase with E_{ac} and I_{ab} is in phase

with E_{ab} since the load is resistance the following diagram will show the proper phase relations.

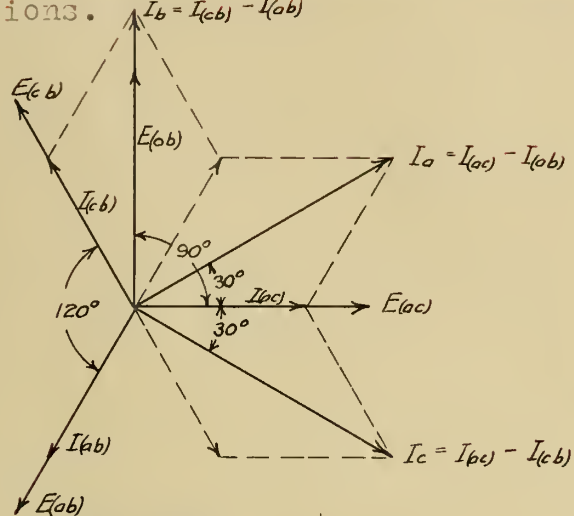


Fig. 85.

An inspection of the above diagram will show that current in transformer coil oa is 30^0 out of phase with the induced voltage in coil oa and that it is leading, while the current in coil oc is lagging by the same angle behind the voltage induced in coil oc. The current in coil ob is in phase with the voltage induced in coil ob. The power output of transformer B is,

$$E\ I = 86.6 \times 100 = 8660 \text{ watts}$$

Power output of coil oa is $E\ I \cos 30^0$

$$= 50 \times 100 \times .866 = 4330 \text{ watts}$$

Power output of coil oc = $E\ I \cos 30^0$

$$= 5- \times 100 \times .866 = 4330 \text{ watts.}$$

Output of transformer A is 4330 + 4330 = 8660 watts which is equal to the output of transformer B. The total output is 8660 + 8660 = 17320 watts, which checks with our original output given by the equation watts = $E\ I\ \sqrt{3}$.

There still remains the question of the greater number

of ampere turns in the secondary of transformer A than in the secondary of transformer B, and if this were true the primary currents of the two transformers would not be equal. Assume transformer B has 86.6 turns on the secondary, then with 100 amperes flowing there would be 8660 ampere turns (active). Assume transformer A has 100 turns and 100 amperes flowing, it would seem that there should be 10000 ampere turns acting on the flux. This is not the case, however, since the current in coil oc is lagging 30° and the current in coil oa is leading 30°. Or we may say that in coil oc there is a wattless component of current tending to demagnetize the core equal to 100 x sin 30° = 50 ampere turns and in the coil oa there is a component of current tending to magnetize the core equal to 100 x sin 30° = 50, the two just neutralize each other and the effective ampere turns will be 100 x cos 30° + 100 x cos 30° = 173 amperes acting on 50 turns or 8660 effective ampere turns which is the same as for transformer B. The primary currents of the two transformers are therefore equal, and the load is equally divided between the two.

The core loss will be the same in these two transformers whether they act as two single phase transformers or are connected for two phase three phase transformation. The I^2R loss of transformer A will, however, be slightly greater when operating under the latter conditions and the efficiency of the combined units will be slightly lower than when operating as single phase units.

X

TRANSFORMERS IN OPEN Δ

Three single phase transformers are often used to transform from a given voltage to some lower or higher voltage as indicated in fig. 86.

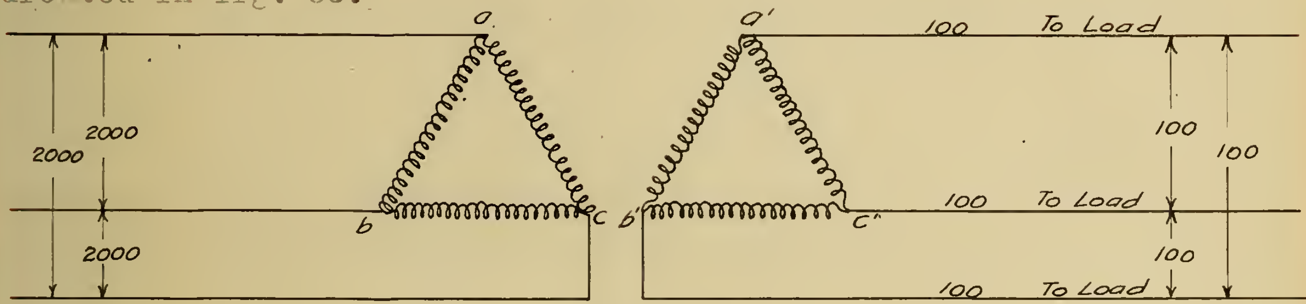


Fig. 86.

Let us assume that the load is balanced and that the line current is 100 amperes. The power output then is, $\sqrt{3}$ E I cos θ. If cos θ is unity then the power is $\sqrt{3}$ x 100 x 100 = 17,300 watts. If transformer coils ac and a'c' are removed from the circuit it is evident that the voltage relations will remain unchanged, see fig. 87.

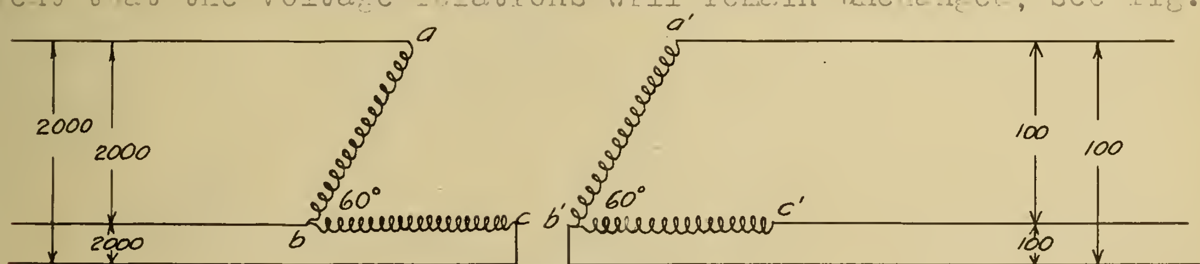


Fig. 87.

In the original case the current in each secondary coil, with 100 amperes in the line is $\frac{100}{\sqrt{3}}$ = 57.8 amperes, which we will assume is full load current for the secondary windings. In the second connection the line current must be the same as the load current and, therefore, can not exceed 57.8 amperes.

The output of the two transformers operating in open

delta will be $\sqrt{3}$ E I cos θ and if cos θ is unity as in the original case the output will be $\sqrt{3}$ x 100 x 57.8 = 10,000 watts, which is a reduction in output of 42.2%, when we might expect the reduction to have been 33 1/3% since one third the transformer capacity was removed.

Again consider the open delta connection as shown in figure 88.

$$-(a) - I_3 + I_1 = 0$$

$$(a) = I_1 - I_3$$

$$-(b) + I_3 - I_2 = 0$$

$$(b) = I_2 - I_3$$

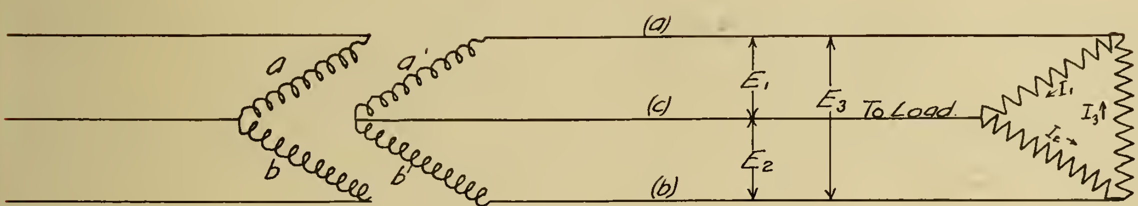


Fig.88.

Let us assume the current supplied to the load lags behind the line voltages E_1, E_2 and E_3 by the angle θ as shown in figure 89.

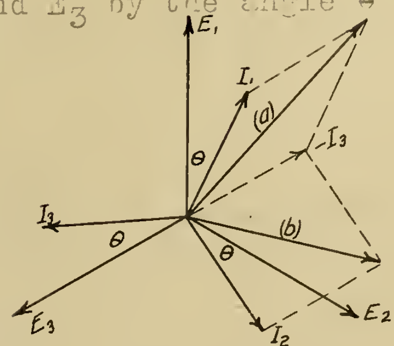


Fig. 89.

The current in line (a) is the vector difference $I_1 - I_3$ and the current in line (b) is the vector difference $I_2 - I_3$. Now since $E_1 = E_2 = E_3 = E$ and since current (a) = current (b), the power

delivered will be,

$$P = E\,a \cos(\theta + 30^\circ) + E\,a \cos(\theta - 30^\circ)$$

$$P = E\,a \;\; \cos(\theta + 30^\circ) + \cos(\theta - 30^\circ)$$

$$= 2\,E\,a \cos\theta \cos 30^\circ$$

$$= 2 \times .866 \times E \times a \times \cos\theta.$$

Assume as in the previous case, that θ = zero. Then the power output of two transformers operating open delta will be, $2 \times .866 \times E \times a$, where E = secondary voltage and a = the coil current. Which shows the output of two transformers operating open delta will be 57.8%, the output of three transformers of the same capacity when operating as in figure 87.

$$\frac{P \text{ open } \Delta}{P \; \Delta} = \frac{2 \times .866 \times E \times a}{1.73\,E \times 1.73\,a} = 57.8\ \%.$$

XI

ALTERNATING CURRENT GENERATOR

The alternating current generator consists of two electrical circuits, i.e. the armature and field. The armature may be either the rotating or stationary type and its windings must be arranged to generate the proper phase voltage and to carry the required amount of current. The previous discussion gives a general idea of the spacing of the windings when polyphase power is desired.

In general the alternator characteristics are much more difficult to predetermine than the characteristics of the transformer and it is impossible to reach the same degree of accuracy in the calculation of this type of machine. However, very good results may be obtained by properly combining the various constants and this chapter will be devoted to the discussion, determination and application of the alternating currong generator constants.

The variables which effect the calculations may be tabulated as follows:

1. Saturation curve characteristics.
2. Friction and windage.
3. Variable value of self inductance of armature windings.
4. Armature resistance.
5. Armature reactions.

In general it may be said that the decrease in terminal voltage is due to three factors, armature resistance, reactance and the demagnetization of the field caused by armature reactions.

The two latter terms are sometimes grouped into one term and designated as "synchronous reactance", which is considered as an ohmic reactance of some predetermined value.

Considering this to be the case the equivalent circuit of the alternator may be reduced to a resistance and reactance connected in series as shown in figure 90.

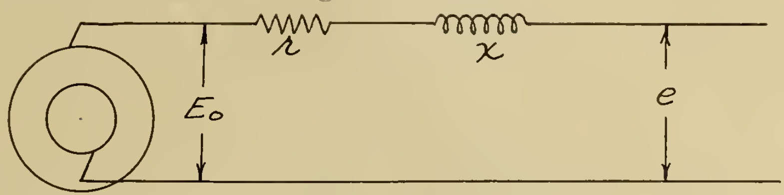


Fig. 90.

Where E_o = total induced voltage,

e = terminal voltage,

r = armature resistance,

x = synchronous reactance.

If we consider these terms as having definite values, the regulation of the machine, operating under various conditions of load may easily be determined since,

$$E_o = e + \dot{I}\dot{Z} = e + (i + j\, i_1)(r + j\, x)$$

$$= e + ir - i_1 x + j\,(i_1 r + i\, x)$$

and the regulation of an alternator is $\frac{E_o - e}{e}$ when expressed in percent.

The following problem will illustrate this method (known as the first approximation).

Problem,

Let r = 10%

x = 40%

e = 100%

Calculate the regulation of the generator, (a) when the load is maintained constant at 100% and the P.F. is varied from 60% lag to 60% lead, (b) when the total current is maintained constant at 100% and the P.F. is varied from 60% lag to 60% lead. Plot regulation curve for each condition of loading.

Solution.

$$E_o = e + \dot{I}\,Z = e + (i + ji_1)(r + j\,x)$$
$$= e + ir - i_1\,x + j(i_1 r + i\,x)$$

If we assume that the power output is constant, then i is constant and equal to unity.

Let I = total current,

i = power component of current,

i_1 = wattless component of current.

TABLE XXI

	Lag			Lead	
P.F.	.60	.80	1.00	.80	.60
I	1.66	1.25	1.00	1.25	1.66
i	1.00	1.00	1.00	1.00	1.00
i_1	-1.33	-.75	1.00	.75	1.33
e	1.00	1.00	1.00	1.00	1.00
$i\ r$	.10	.10	.10	.10	.10
$i_1 x$	-.532	-.30	0	.30	.532
$e + i\ r - i_1 x$	1.632	1.40	1.10	.80	.568
$(e + i\ r - i_1 x)^2$	2.66	1.96	1.21	.64	.268
$i_1 r$	-.133	-.075	0	.075	.133
$i\ x$	.40	.40	.40	.40	.40
$i_1 r + i\ x$	.267	.325	.40	.475	.533
$(i_1 r + i\ x)^2$	.0713	.1058	.16	.226	.284
E_o	1.65	1.44	1.17	.930	.742
Reg.	.65	.44	.17	-.07	-.258

Regulation Constant Total Current I = 1.00

TABLE XXII

P.F.	.60	.80	1.0	.80	.60
I	1.00	1.00	1.00	1.00	1.00
i	.60	.80	1.00	.80	.60
i_1	.80	.60	0	.60	.80
e	1.00	1.00	1.00	1.00	1.00
$i\ r$	.06	.08	.10	.08	.06
$i_1 x$	-.32	-.24	0	.24	.32
$e + i\ r - i_1 x$	1.38	1.32	1.10	.84	.64
$(e + i\ r - i_1 x)^2$	1.905	1.744	1.21	.706	.409
$i_1 r$	-.08	-.06	0	.06	.08
$i\ x$	.24	.32	.40	.32	.24
$i_1 r + i\ x$	.22	.26	.40	.38	.32
$(i_1 r + i\ x)^2$	.048	.067	.16	.144	.102
E	1.397	1.345	1.17	.922	.714
Reg.	.397	.345	.17	-.078	-.286

Tables XXI and XXII have been derived from the above theory and the results of these tables may be seen plotted to scale on curve sheet 20, figure 90.

A second method of calculating the regulation of an alternator is based on the following assumptions.

It has been found that the equation of the saturation curves of most alternators is very closely approximated by the following,

$$E = \frac{K\ i}{1 + K_1 i}$$

where E = the no load terminal voltage and i is the field current, K and K_1 being constants of the magnetic circuit. Assuming this equation to be true we may find the saturation curve for any alternator provided two points on the curve are known.

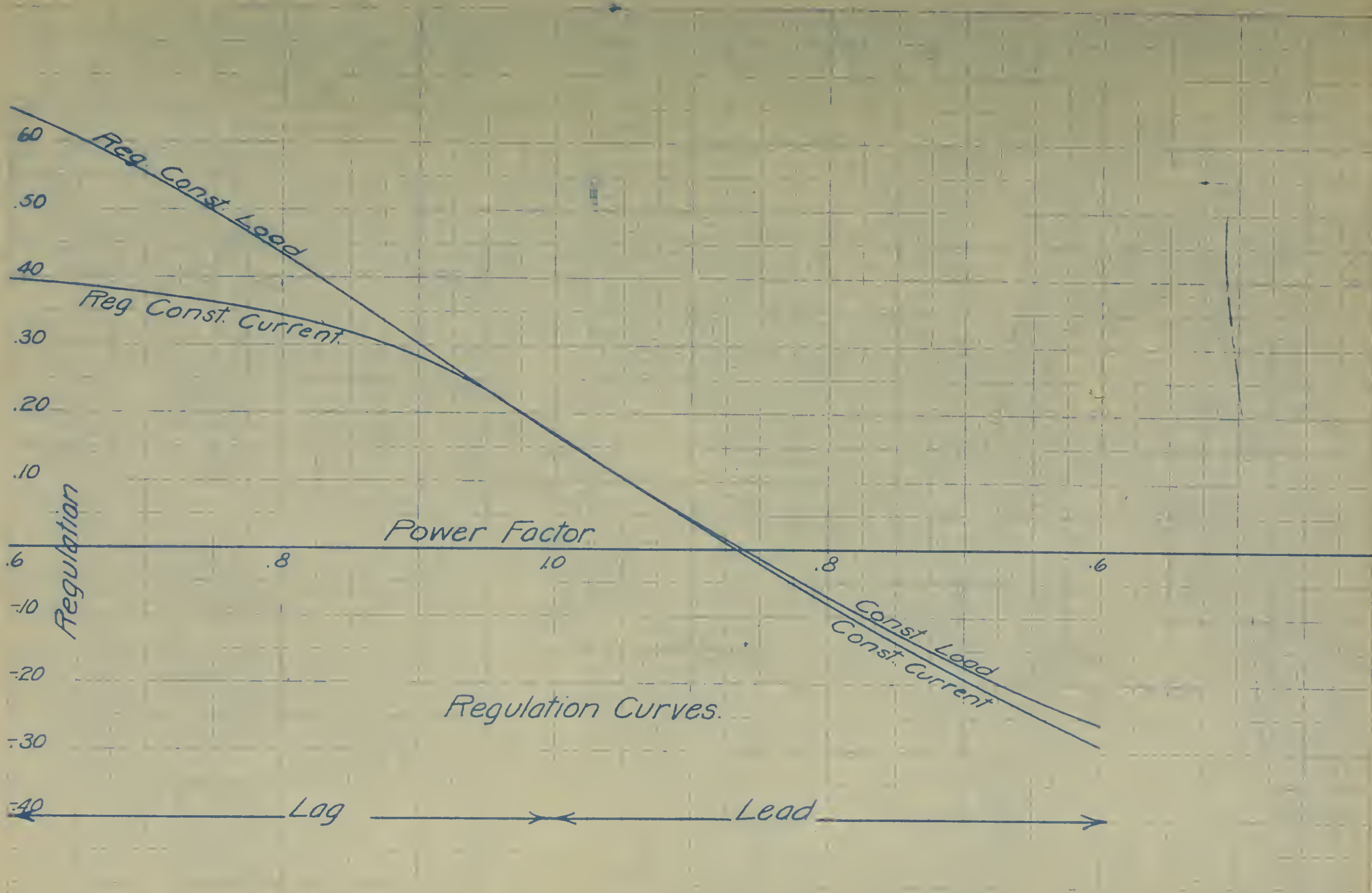


Fig. 90 a

Curve Sheet 20.

Let us take the following data,

$E = 1 \qquad i = 1$

$E = .6 \qquad i = .5$

from which data we find $K = 1.5$ and $K_1 = .5$. Then for the following values of i we may determine E and the saturation curve as shown in Table XXIII.

TABLE XXIII

i	E
.4	.500
.5	.600
.6	.692
.7	.778
.8	.851
.9	.931
1.0	1.000
1.1	1.064
1.2	1.125
1.4	1.235
1.6	1.333
1.8	1.420

The curve figure 91 is plotted from the above data and represents the open circuit voltage plotted against field current for an alternating current generator. The values, of course, are express in percent of normal values.

Now let F = the field excitation at no load necessary to give normal voltage and let F_o = the field excitation at full load to give normal voltage. Assume $F_o = \frac{E_o}{E} \times F$ which would be absolutely correct if the saturation curve was a straight line passing through the origin.

To find the regulation of an alternator by the second method, calculate E_o as in method number one.

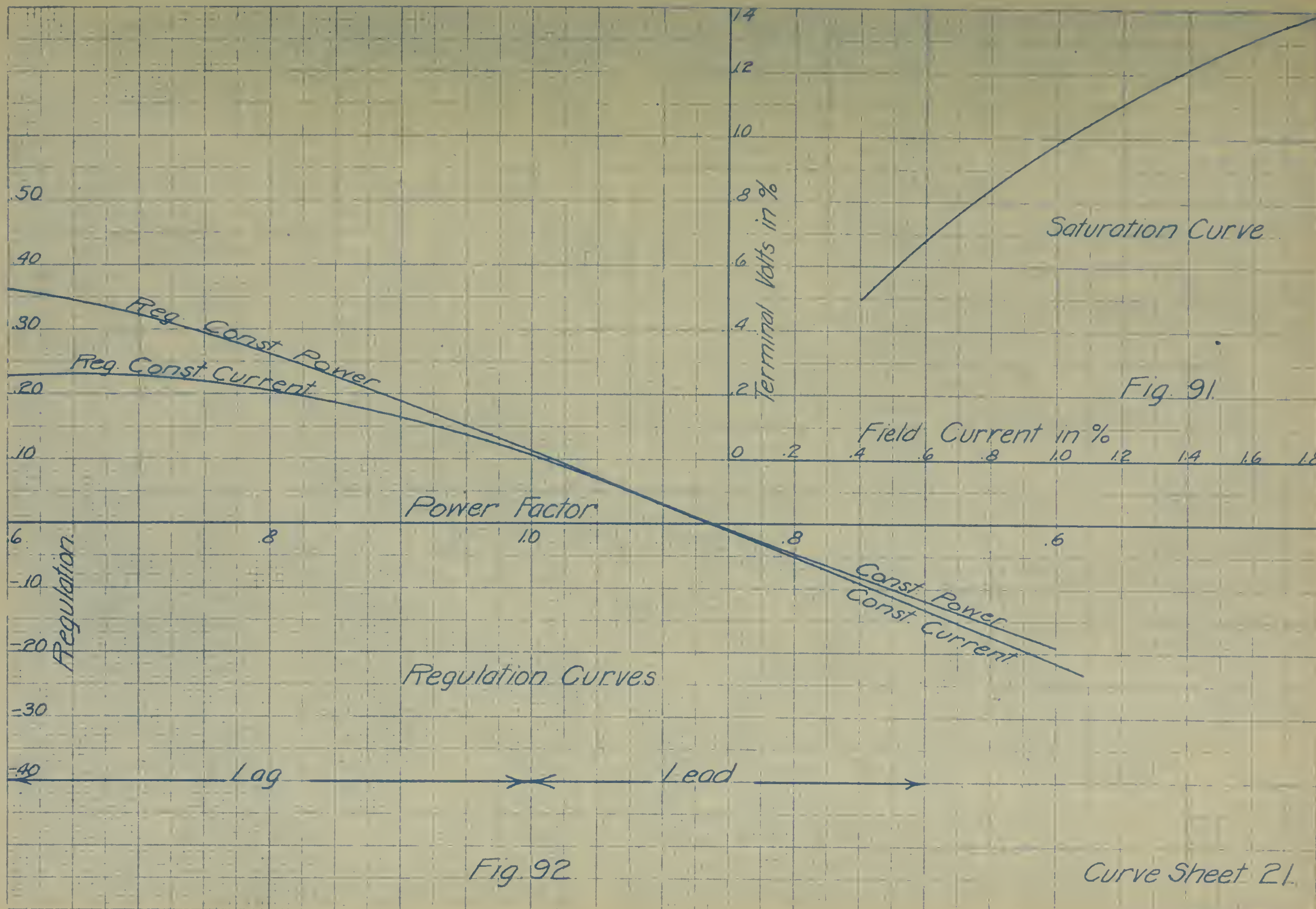


Fig. 91.

Fig. 92

Curve Sheet 21.

From the previous data it is seen that the ratio $\frac{F}{E}$ is unity, hence for any calculation $F_o = E_o$. Having found F_o, from the saturation curve we may find the corrected value of E_o or E'_o which is more nearly the correct value of E_o. The regulation will then be $\frac{E'_o - E}{E}$.

The second method gives values of regulation somewhat lower than the first and it is probable that the first method gives too great a regulation while the second gives values which are too small. Table XXIV shows the results of calculating the original machine first with constant power output and varying powerfactor, and second with constant total current output and varying power factor.

The results of this table are seen plotted to scale on curve sheet 21, figure 92.

TABLE XXIV

Constant Power.

	P.F.	E_o	F_o	E'_o	Reg.
Lag	.6	1.65	1.65	1.36	.36
Lag	.8	1.44	1.44	1.26	.26
	1.0	1.17	1.17	1.11	.11
Lead	.8	.93	.93	.955	-.045
Lead	.6	.742	.742	.81	-.19

Constant Current.

Lag	.6	1.397	1.397	1.23	.23
Lag	.8	1.345	1.345	1.205	.205
	1.0	1.170	1.170	1.11	.11
Lead	.8	.922	.922	.95	-.05
Lead	.6	.714	.714	.795	-.205

Before taking up the third and last method of voltage regulation it is necessary to study the effect of armature reac-

tions on the operation of single and polyphase machines. For the sake of simplicity we will consider concentrated windings, (i.e.) the windings of one phase bunched together in one slot.

The following discussion of armature reaction will be based on the above assumptions.

Armature Reactions. Single Phase.

Consider the winding of a single phase machine concentrated as shown in figure 93 . The equation for the voltage is,

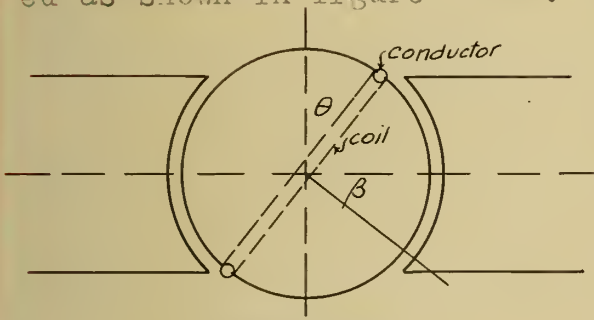


Fig. 93

$$e = E \sin \theta,$$

and if we assume the power factor = $\cos \alpha$, then $i = I \sin (\theta - \alpha)$, and since the magneto-motive-force (m.m.f.) is the produce of the current and turns, T, through which it flows it is evident that,

$$\text{m. m. f.} = I\ T \sin (\theta - \alpha)$$

and the effective component or component which is parallel to the field will be,

$$I\ T \sin (\theta - \alpha) \cos$$

and since,

$$\beta = \theta \quad \text{Eff. m.m.f.} = I\ T \sin (\theta - \alpha) \cos \theta$$

which is a quantity having double the frequency of the angle θ.

The average value of the effective m.m.f. will now be found.

$$\text{Av. eff. m.m.f.} = \frac{1}{2\pi} \int_0^{2\pi} I\ T \sin (\theta - \alpha) \cos \theta \, d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} I\ T \sin\theta \cos\alpha \cos\theta - \frac{1}{2\pi}\int_0^{2\pi} I\ T \sin\alpha \cos^2\theta\, d\theta$$

Since α is assumed as constant we may write,

$$\text{Av.eff. m.m.f.} = \frac{\cos\alpha}{2\pi} I\ T \int_0^{2\pi} \sin\theta \cos\theta - \frac{\sin\alpha}{2\pi} I\ T \int_0^{2\pi} \cos^2\theta\, d\theta$$

$$= -\frac{1}{2} I\ T \sin\alpha \text{, for a single phase generator.}$$

If the power factor is unity the average eff. m.m.f. is,

$$\frac{1}{2\pi}\int_0^{2\pi} I\ T \sin\theta \cos\theta\, d\theta = 0$$, from which it is seen that there is no average demagnetizing component of armature reactions when the power factor is unity.

Now let us consider the polyphase winding and for convenience we will use the three phase armature, the windings being displaced 120 electrical degrees. Let us assume that the e.m.f. of the first phase is as illustrated in figure 93 , or

$e_1 = E \sin\theta$ then $i_1 = I \sin(\theta - \alpha)$

$e_2 = E \sin(\theta - 120^\circ)$ $i_2 = I \sin(\theta - \alpha - 120^\circ)$

and $e_3 = E \sin(\theta - 240^\circ)$ $i_3 = I \sin(\theta - \alpha - 240^\circ)$

The effective component of phase #1 will be,

m.m.f. $= I\ T \sin(\theta - \alpha)\cos\theta$, for phase #2,

m.m.f. $= I\ T \sin(\theta - \alpha - 120^\circ)\cos(\theta - 120^\circ)$, and for phase #3 m.m.f. $= I\ T \sin(\theta - \alpha - 240^\circ)\cos(\theta - 240^\circ)$, and the total m.m.f. will be the sum of all these values at any instant.

The average eff. m.m.f. will therefore be,

$$\text{Av. eff. m.m.f.} = \frac{1}{2\pi}\int_0^{2\pi} I\ T \sin(\theta - \alpha) \cos\theta \, d\theta + \frac{1}{2\pi}\int_0^{2\pi} I\ T \sin(\theta - \alpha - 120^\circ)$$

$$\cos(\theta - 120^\circ)\, d\theta + \frac{1}{2\pi}\int_0^{2\pi} I\ T \sin(\theta - \alpha - 240^\circ) \cos(\theta - 240^\circ)\, d\theta$$

$$= -\frac{3}{2} I\ T \sin\alpha$$ or for an n phase machine,

$$\text{m.m.f.} = -\frac{n}{2} I\ T \sin\alpha.$$

Table XXV and curve sheet number 22 show the variation of the demagnetizing component and shifting component of armature reactions from instant to instant for one complete cycle. It is evident from the figure that the shifting component, which is represented by $\sin^2\theta$, has an average value of .5 and that the average value of the demagnetizing component $\sin\theta\cos\theta$ is zero, therefore the total average armature reaction with unity power factor is equal to .5.

TABLE XXV

(1)	(2)	(3)	(4)	(5)	(6)
θ	Sin θ	Cos θ	(2) x (3)	$\sin^2\theta$	(4) + (5)
0	0	1.00	0	0	0
20	.342	.939	.321	.117	.438
40	.642	.766	.484	.412	.896
60	.866	.500	.433	.750	1.180
80	.984	.173	.171	.968	1.139
100	.984	-.173	-.171	.968	.797
120	.866	-.500	-.433	.750	.317
140	.642	-.766	-.484	.412	-.072
160	.342	-.939	-.321	.117	-.204
180	0	-1.00	0	0	0
200	-.342	-.939	.321	.117	.438
220	-.642	-.766	.484	.412	.896
240	-.866	-.500	.433	.750	1.180
260	-.984	-.173	.171	.968	1.139
280	-.984	.173	-.171	.968	.797
300	-.866	.500	-.433	.750	.317
320	-.342	.766	-.484	.412	-.072
340	-.342	.939	-.321	.117	-.204
360	0	1.00	0	0	0

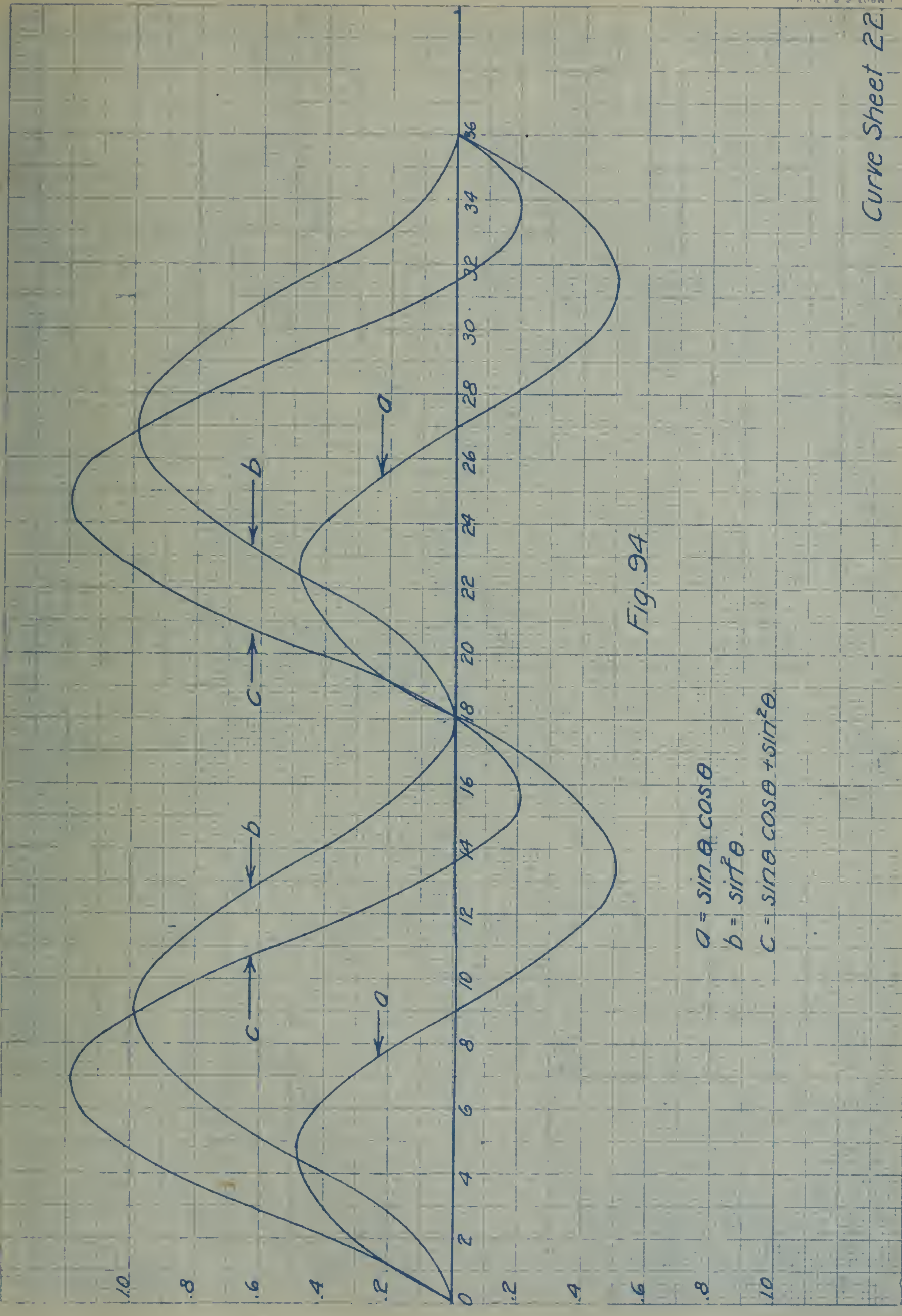

a = sin θ cos θ
b = sin²θ
c = sin θ cos θ + sin²θ
a
b
c
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36
1.0 .8 .6 .4 .2 0 .2 .4 .6 .8 1.0
Curve Sheet 22

Fig. 94

Another point of interest is found in a study of the effectiveness of the demagnetizing component as compared with the shifting component of armature reactions.

Figures 95, 96 and 97 show the various positions of the armature coils for maximum current with different power factors, and a study of these figures will at once show the action of armature m.m.f. on the field magnets. Again, it may be seen that the reluctance of the magnetic path of the armature flux in figure 95 is much greater than in either of the two following figures. From which we may draw the conclusions that the power or energy current is not as effective as the wattless current in producing flux in the armature.

It is also evident that the self-inductance of an armature winding in position as shown in figure 95 is greater than in either of the two following positions, and hence, the power component of current produces a greater reactive drop in the armature than is produced by the wattless current.

The total field impressed must therefore be the vector sum of the resultant field and the armature reactions.

Having the saturation curve for a particular machine we may proceed to calculate the field characteristics as follows. From the diagram it is evident that the value of n in ampere turns corresponds to the voltage $e + i\ r - i_1x$, since this flux or m.m.f. is 90^0 in advance of $e + i\ r - i_1x$. Therefore for any calculated value $e + i\ r - i_1x$ we may find the corresponding value of n directly from the saturation curve, since this is the real component

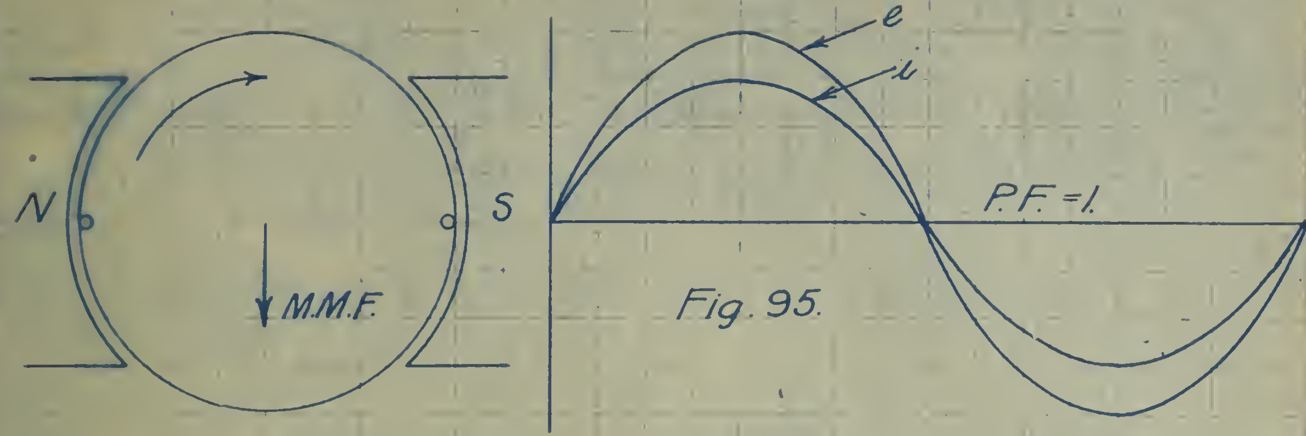


Fig. 95.

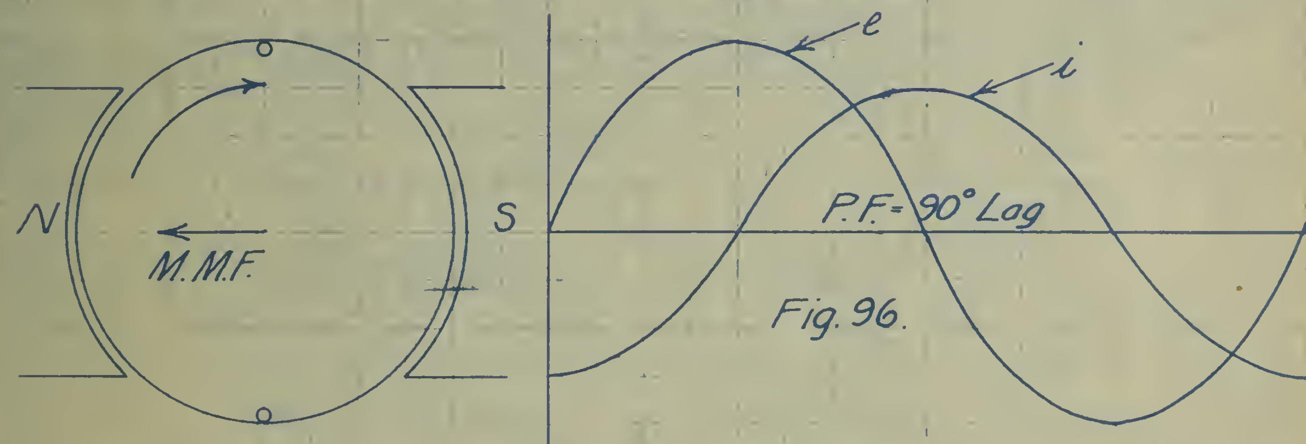


Fig. 96.

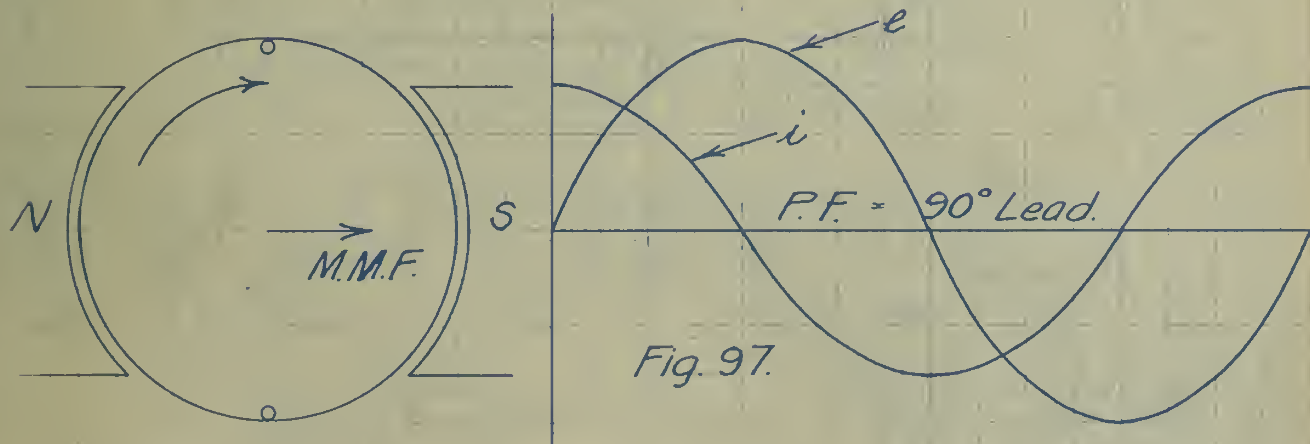


Fig. 97.

Curve Sheet 23.

of voltage or is the voltage which would exist at the terminals of the machine on open circuit with the field excited to a value corresponding to n, and N = armature turns per pole per phase.

Since $1\ 1/2 \sqrt{\ }$ N is a constant for any particular machine we may call this term M and the armature reactions then reduce to I M. But $I = i + j\, i_1$, and we may write,

$$\text{Arm. reactions} = i\, M + j\, i_1 M.$$

This would be the proper way to handle the armature reactions of a machine of the round rotor type, but it has already been shown that when a machine has definite pole frame the effectiveness of the energy component of current is not as great as the wattless component. Therefore it is necessary to have two values of M, (i.e.) m and m_1 and our equation for armature reactions becomes,

$$\text{Arm. reactions} = i\, m + j\, i_1\, m_1\ ,$$

the flux produced the resultant m.m.f. and this m.m.f. is the vector sum of the impressed field m.m.f. and the armature m.m.f. or armature reactions as is shown in the diagram.

Let the resultant m.m.f. be resolved into two components at 90^0 to each other the vertical component n and the horizontal component n'. Then for the general expression we may write,

$$\text{Resultant m.m.f.} = j\, n + n'.$$

Now let the field m.m.f., necessary to overcome the armature reactions, be resolved in a like manner into two components as represented in the diagram. Then the armature reaction m.m.f. $= j\, m_1 + m$.

It has already been shown that the armature reactions of a three phase alternator may be represented by the equation $1\ 1/2 \sqrt{2}\ I\ N$.

I = eff. current per phase.

In the third method of calculating armature reactions we will not consider the armature reactance and armature reactions grouped together, but will treat each as a separate quantity.

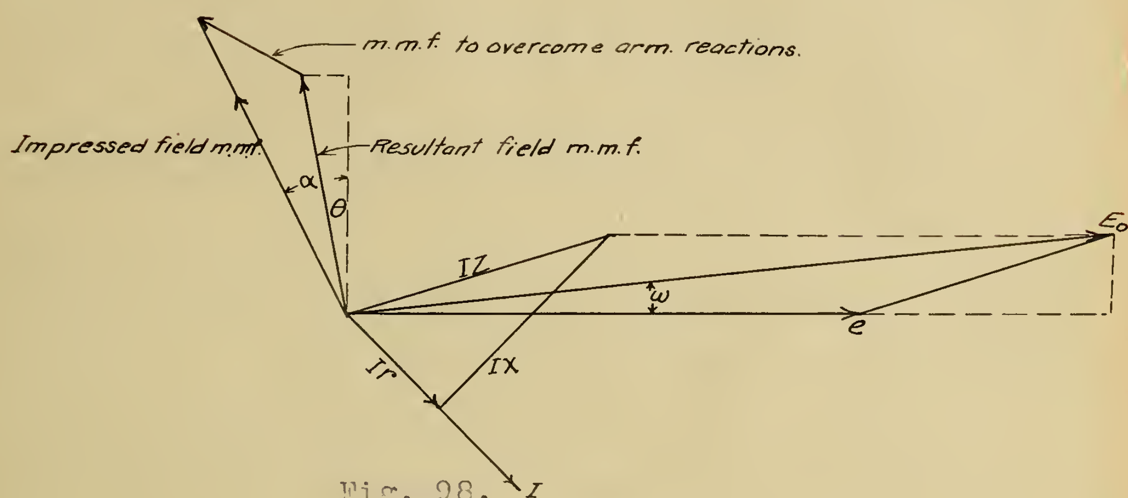


Fig. 98.

In the figure 98 let e represent the terminal e.m.f. per phase. Then if x is the internal reactance per phase and r is the armature resistance per phase it is evident that,

$$E_o = e + \dot{I}\, Z = e + (i + j\, i_1)\ (r + j\, x)$$
$$= e + ir - i_1x + j\ (ix + i_1r)$$

Since E_o is the induced e.m.f. it is evident that the resultant field flux and hence the resultant field m.m.f. must be 90^o in advance of E_o.

The resultant field flux is of course the vector sum of the armature reactions and the field ampere turns. Again, from the diagram we see that the angle θ = angle ω and therefore the following relation must exist.

$$n' = n\ \left(\frac{i\ x + i_1r}{e + ir - i_1x}\right),$$

and in order to keep all terms in the proper phase position it is necessary to write,

$$n' = -n\left(\frac{i\ x + i_1 r}{e + ir - i_1 x}\right).$$

It is now evident that the total m.m.f. impressed on the field must be the vector sum of $jn + n'$, and $j\ i_1\ m_1 + i\ m$. If these two quantities are added it will be found that $m\ i$ will be added to n which is not correct for the diagram. This is due to the fact that, since, the armature reactions are expressed by $i\ m + j\ i_1\ m_1$, we must express the field m.m.f. to overcome the armature reactions by $j^2\ (i\ m + j\ i_1\ m_1)$, which in reality is rotating the armature reaction vector through 180^0.

The total resultant field is now,

$$(jn + n') - (mi + j\ i_1\ m_1)$$

and since n' is always negative it is seen that the shifting component of the total field m.m.f. is,

$$-n' - i\ m = -(n' + i\ m)$$

and the demagnetizing component is $jn - j\ i_1 m_1 = j(n - i_1\ m_1)$, these quantities add if i_1 is a lagging current and subtract if i_1 is a leading current which is the correct action of such armature currents.

The following problem has been solved by applying the above theory, two types of machines are used and their field characteristics plotted.

100 K.W., 2300 v. - y - 3-phase.

(1330 v. per phase.) 25 amp. per phase.

Armature reactions 1490 A.T. total.

Constants (per phase)

Definite pole

$m_1 = 59.5$

$m = 40$

$r = .69$ ohms

$x_1 = 5.8$ ohms

$x = 8.7$ ohms

Round rotor.

$m = m_1 = 59.5$

$r = .69$ ohms

$x = x_1 = 8.7$ ohms

Saturation Curve.

$A.T._f$	$E_{ter.}$
400	250
1000	700
2000	1470
3000	2060
3650	2300
5000	2700
6000	2900

Keeping the energy current constant vary the wattless current through the following values, -37.5, -25, 0, 25, 37.5. Calculate field characteristics and regulation.

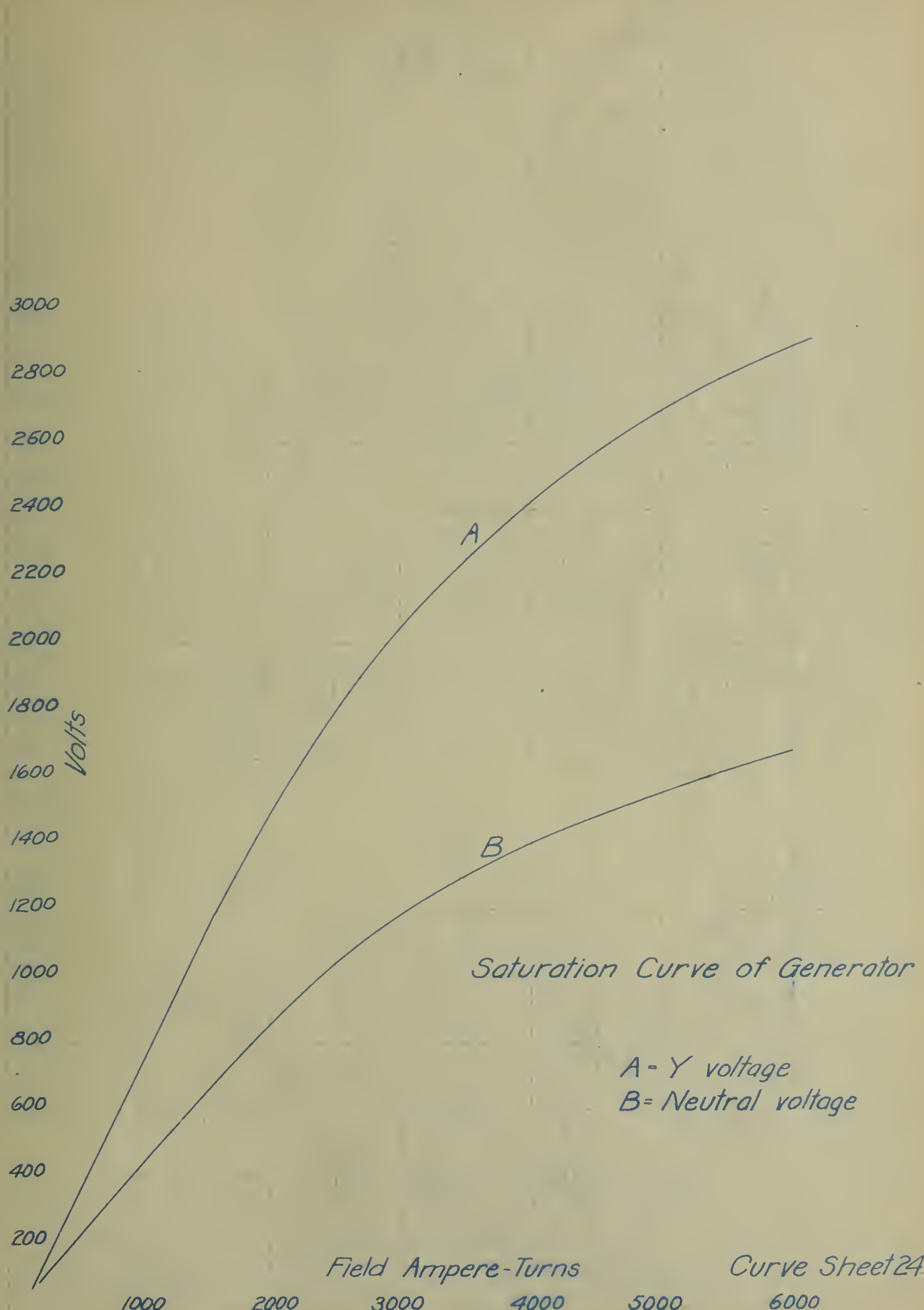

3000
2800
2600
2400
2200
2000
1800
1600
1400
1200
1000
800
600
400
200
Volts
A
B
Saturation Curve of Generator
A = Y voltage
B = Neutral voltage
Field Ampere-Turns
Curve Sheet 24
1000
2000
3000
4000
5000
6000

TABLE XVI

e	1330	1330	1330	1330	1330
i	25	25	25	25	25
i_1	-37.5	-25	0	25	37.5
$i\ r$	17.25	17.25	17.25	17.25	17.25
$i_1 x_1$	-217.5	-145.0	0	145	217.5
$e + i\ r - i_1 x_1$	1564	1492	1347	1202	1130
n	5050	4510	3700	3025	2780
$i\ x$	217.5	217.5	217.5	217.5	217.5
$i_1 r$	-25.9	-17.25	0	17.25	25.9
$i\ x + i_1 r$	191.6	200.	217.5	234.7	243.4
$a = \frac{i\ x + i_1 r}{e + i r - i_1 x}$	.1225	.134	.161	.195	.215
$n' = -n \times a$	-619	-604	-596	-591	-598
$i_1 m_1$	-2231	-1488	0	1488	2231
$i\ m$	+1000	+1000	+1000	+1000	+1000
$n - i_1 m_1$	7281	5998	3700	1537	529
$b = (n - i_1 m_1)^2 \div 10^6 =$	53.0	36.	13.7	2.36	.28
$n' - i\ m$	-1619	-1604	-1596	-1591	-1598
$c = (n' - i\ m)^2 \div 10^6 =$	2.62	2.57	2.55	2.53	2.56
$b + c$	55.62	38.57	16.25	4.89	2.84
$b + c$	7.45	6.21	4.03	2.21	1.68
$1000 \times b + c$	7450	6210	4030	2210	1680
I_{total}	45.1	35.3	25	35.5	45.1
tan	.222	.267	.431	1.035	5.05
	$12^{\circ}31'$	$14^{\circ}57'$	$23^{\circ}19'$	$46^{\circ}0$	$71^{\circ}51'$

TABLE XVII

e	1330	1330	1330	1330	1330
i	12.5	12.5	12.5	12.5	12.5
i_1	-37.5	-25	0	25	37.5
$i\ r$	8.62	8.62	8.62	8.62	8.62
i_1x_1	-217.5	-145	0	145	217.5
$e + i\ r - i_1x_1$	1556	1483	1338.6	1193	1121
n	4960	4460	3650	3000	2750
$i\ x$	108.7	108.7	108.7	108.7	108.7
i_1r	-25.9	-17.25	0	17.25	25.9
$i\ x + i_1r$	82.8	91.5	108.7	125.9	134.6
$a = \frac{i\ x + i_1r}{e + i\ r - i_1x}$	.0532	.0617	.0812	.1055	.120
$n' = -n \times a$	-264.0	-275.3	-296.3	-316.3	-330.0
i_1m_1	-2231	-1488	0	1488	2231
$i\ m$	500	500	500	500	500
$n - i_1m_1$	7191	5948	3650	1512	519
$b = (n - i_1m_1)^2 \div 10^6$	51.7	35.4	13.36	2.286	.269
$n' - i\ m$	-764.0	-775.3	-796.3	-816.3	-830.0
$c = (n' - i\ m)^2 \div 10^6$	.584	.600	.628	.667	.689
$b + c$	52.28	36.0	13.98	2.953	.958
$b + c$	7.23	6.0	3.74	1.718	.309
$1000 \times b + c$	7230	6000	3740	1718	309.0
I_{total}	40.2	27.9	12.5	27.9	40.2
tan					

TABLE XVIII

e	1330	1330	1330	1330	1330
i	0	0	0	0	0
i_1	-37.5	-25	0	25	37.5
$i\ r$	0	0	0	0	0
i_1x_1	-217.5	-145	0	145	217.5
$e + i\ r - i_1x_1$	1547.5	1475	1330	1185	1112.5
n	4940	4430	3600	2950	2700
$i\ x$	0	0	0	0	0
i_1r	-25.9	-17.25	0	17.25	25.9
$i\ x + i_1r$	-25.9	-17.25	0	17.25	25.9
$a = \frac{i\ x + i_1r}{e + ir - i_1x}$	.0617	.0117	0	.0145	.0233
$n' = -n \times a$	-82.5	-51.8	0	-42.8	-63.0
i_1m_1	-2231	-1488	0	1488	2231
$i\ m$	0	0	0	0	0
$n - i_1m_1$	71.71	5918	3600	1462	469
$b = (n - i_1m_1)^2 \div 10^6$	51.4	35.0	13.0	2.14	.22
$n' - i\ m$	-82.5	-51.8	0	-42.8	-63.3
$c = (n' - i\ m)^2 \div 10^6$	.0068	.00268	0	.00183	.004
$b + c$	51.407	35.003	13	2.133	.224
$b + c$	7.172	5.92	3.6	1.456	.1495
$1000 \times b + c$	7172	5920	3600	1456	149.5
I_{total}	37.5	25	0	25	37.5

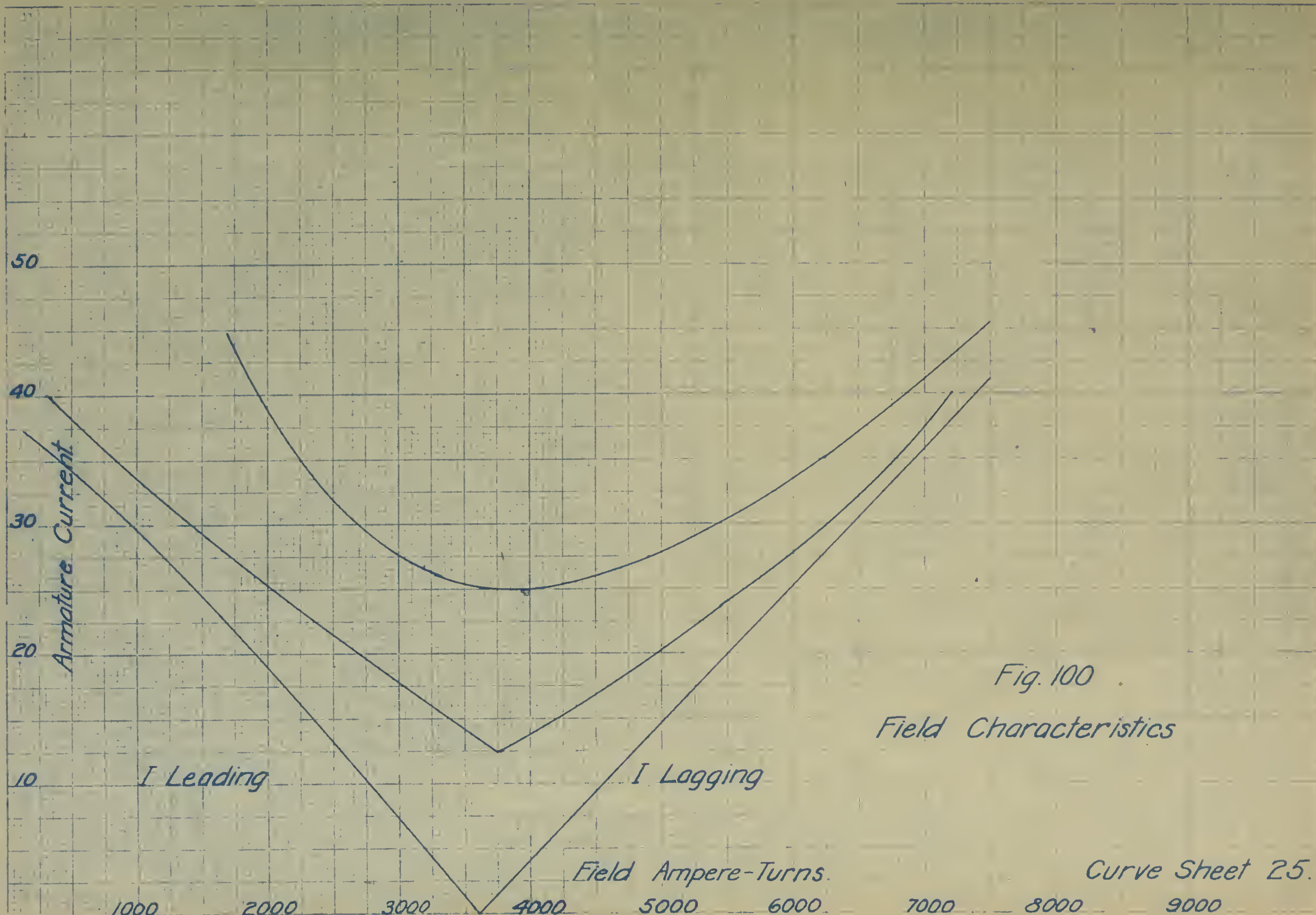


Fig. 100.

Field Characteristics

Curve Sheet 25.

XII

CONCLUSIONS

By use of the principles set forth in this thesis calculations of the behavior of various electrical machines may be readily made. It must, of course, be understood that a complete study of any one machine is not possible in this thesis.

The behavior of the machines which have been considered may, however, be studied in greater detail by the proper application of the theory as outlined, while the actual work given represents one semesters study in Alternating Current Theory.